GEORGIA DEPARTMENT OF REVENUE

LOCAL GOVERNMENT SERVICES DIVISION

ADVANCED INCOME

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INCOME & EXPENSE STATEMENT

Reconstructed from owner's statement - Often the appraiser will use the information from the owner's operating statement and reconstruct an income statement for the property, based on the current market rents and typical proper expenses for one year.

The reconstructed income & expense statement (operating statement) is developed from rent comparables in the area.

Format of income and expense statement:

- Gross income estimate (potential gross income - PGI)
- Potential gross income is the annual economic rent for the property at 100 percent occupancy.
- It includes the market rent from all scheduled sources.

Types Of Rents

Market rent - The rental income that a property would most probably command in the open market; indicated by current rents paid and asked for comparable space as of the date of the appraisal.

> Market rent should reflect the location of the property, size of the property, supply and demand factors, and terms of lease between knowledgeable and prudent owners and tenants and is also known as economic rent.

Contract rent - The actual rental income specified in a lease; actual rent payable as distinguished from market rent, or rent that could be expected if the property were available for rent on the open market.

Minimum base rent - The fixed portion of the rent under the terms of a percentage lease.

Minimum rent - Minimum rent is closely associated with the concept of return on and of the investment. Individual tenants may pay varying rates per square foot of occupied space; however, the income from all tenants is the amount that the developer/owner must have to satisfy debt and equity positions of the fee simple estate of the entire property.

Percentage rent - Percentage rent is rental income received in accordance with the terms of a percentage clause in a lease. Percentage rent is typically derived from rental tenants and is based on a certain percentage of their sales revenue.

Overage rent - the percentage rent paid over and above the guaranteed minimum rate; payable under the terms of a lease that requires additional rent under prescribed conditions such as the achievement of a particular level or profit of retail sales volume by the tenant; commonly used in connection with additional rent paid under a percentage clause. (Illustration 2-1)

Leasehold rent (leasehold income) - Occurs when market rent is greater than contract rent. In effect, the tenant is receiving as income the amount of the difference between contract rent and market rent.

Excess rent - the amount of which contract exceeds market rent at the time of the appraisal; created by a favorable lease and may reflect locational advantage, unusual management, or lease execution in an earlier, stronger rental market. Due to higher risk inherent in the receipt of excess rent, it may be calculated separately and capitalized at a higher rate in the income capitalization approach. (Illustration 2-2)

Types of Rentable Area

Gross building area - The total constructed area of a building as measured on the outside of the structure.

Gross Leasable area (GLA) -All area within the outside walls, including lobbies, washrooms, janitor's closets, and so on, but excluding building stairs, fire towers, elevator shafts, flues, vents, stacks, pipe shafts, and vertical ducts if they serve more than one floor.

Net Leasable area (NLA) - The area within a building or structure that is actually occupied by an individual tenant. Net Leasable area does not include any of the common areas, such as lobbies and restrooms, shared by other tenants and is also referred to as net rentable area (NRA).

Rent Concessions

A rent concession is a loss in income for the owner because of an agreement with the tenant to either reduce the rent on short term basis, or allow free rent for a short term, or supply higher than normal tenant improvement (TI) work.

- The concession calculation and subsequent deduction can only be made if this type of concession is a standard market practice. It may be that the owner has a need to fill vacant space as soon as possible, and elects to handle it by granting concessions, when in fact, the typical landlord does not. Adjusting for atypical rent concession will provide an value estimate of the leased fee value, not fee simple value.
- The appraiser needs to research the current market concerning concessions before making decisions on when, or if, it is appropriate to adjust the income stream.
- If the rent concession is considered an appropriate deduction because of what the market indicates, it should be removed from the income in a

manner similar to a vacancy deduction. However, if the concession being given appears to be short-term in nature, a suggestion for a more appropriate way to address that income loss would be to deduct the lost rent by discounting it over the projected short term, then deducting that total from the final income estimate. This loss can be calculated two ways. Illustrations 2-3 & 2-4

Vacancy & Collection Loss Derived From the Market

Vacancy and credit losses are income lost due to vacancies and/or tenants defaulting on lease payments. Typically, all properties will experience some amount of vacancy during their economic life. The amount that is allowed for vacancy and collection loss must be based on an analysis of properties that are similar to the subject.

- Forecasting vacancy losses means analyzing how competitive your rents are within the local market. The impact of the supply of comparable space and quantity of space demanded by tenants is an important component in the vacancy factor. Leases may also include options to renew under different terms than the present lease.
- Credit losses may be incurred when tenants are in arrears on their rent, as an example, when a tenant eviction or bankruptcy proceedings are pending, etc. Therefore, collection loss would be any circumstance that may result in a loss of rental income and/or additional collection and legal expense.

Absorption or Below Market Occupancy Levels

- Absorption is a term used when a property has a vacancy rate greater than the market vacancy rate as of the date of the appraisal.
- Absorption would normally occur in new construction, such as offices, retail, and multi-family improvements and where obtaining a normal occupancy derived from market analysis will occur over time through

absorption. Furthermore, this vacancy problem would need to be considered a temporary problem to be classified as abnormal vacancy.

- The calculations for this consist of using some advanced present value procedures that are demonstrated in the Discounted Cash Flow section. A more detailed explanation and Examples are included.

Miscellaneous Income

Miscellaneous income may come from several sources other than the scheduled rents. Miscellaneous income covers all income generated by the operation of the real property that is not derived directly from scheduled rental of space. Some examples of miscellaneous income are:

a. parking fees,
b. vending machines,
c. coin operated laundries
d. cable installation

This income may or may not be attributable to the real property. The appraisers must consider this as business income, or as income to the real property, depending on the market.

Effective Gross Income (EGI)

Effective gross income is obtained after determining the potential gross income for the property. EGI is PGI less vacancy and collection loss plus appropriate miscellaneous income.

Allowable Operating Expenses

Operating expenses are ordinary and typical expenses that are necessary to keep the property functional and rented competitively with other properties in the area.

- Operating expenses vary from property to property, depending on the type of occupancy, use type, and quality of management.
- In analyzing the operating expenses for a property, the operating statements for the subject must be reviewed and the following considered:
    a. Does the expense amount appear to be typical for the property in question, and is the amount substantiated by expense statements of comparable properties?
    b. Do the expenses tend to appear infrequently?
    c. Do the expenses appear to indicate typical management?
    d. Do the expenses indicate a specific weakness of the property in question?
    e. Are the various reported expenses consistent as they relate to each other (maintenance, age, and replacement reserves)?
    f. Is the ratio of expenses to effective gross income comparable to those for competitive properties?

Examples of proper expenses are:

- Management
- Insurance
- Utilities

- Maintenance
- Legal and accounting fees
- Employee wages and fringe benefits
- Yard care
- Miscellaneous expenses
- Replacement Reserve

Replacement Reserve - An allowance in an annual operating statement for replacement of short-lived items that will not last for the remaining economic life of a property.

The recognition of an annual expense, used to generate the NOI, which may not have yet occurred.

Short-lived items - are items which are expected to have a remaining economic life less than the economic life of the property.

- *The amount of expense for short-lived items is computed by dividing the replacement cost of the item by the number of years of total economic life.*

Some examples of 'Replacement Reserves' are carpets, drapes, heating, air conditioning, ranges, refrigerators, water heaters, and paving, etc.

EXPENSE REIMBURSEMENTS & CAM EXPENSES, PASS THROUGH EXPENSES & EXPENSE STOPS

Expense reimbursements:

When a tenant is forced to pay the landlord additional expenses because of the expense stop, that amount of money is normally classified as expense reimbursements and shows up as income in the operating statement.

The expense portion of the operating statement will probably reflect all expenses paid by the landlord, including that amount reimbursed. The appraiser

needs to use care when using this information when analyzing a specific property's income and expense data.

Reimbursements may include CAM (see below) expenses being paid by the tenant. The operating statement should be checked to see if these expenses are showing up as an expense item or not.

CAM Expenses

CAM (common area maintenance) expenses are usually called out in a lease for the purpose of defining a specific cost that the tenant will be responsible for.

This type of "called out" expense is more common with retail leases than other property type. Newer retail property tenants will ordinarily pay all rented space expenses. This would leave the landlord paying all operating costs for the common area. Many landlords will pass this expense on to the tenant by setting that item up in the lease as something the tenant must pay the landlord, usually on a prorated basis (tenant space to total area ratio).

Pass-through Expenses

This is a term used for those expenses the property incurs that exceed the level of expense the landlord is willing to pay. This expense is then paid by the tenant, usually on a prorated basis according to its space percentage of the total area.

Expense Stop(s)

- This is a term used for the level of expense that the landlord is willing to pay. This number is usually expressed as a dollar amount per square foot, and includes all expenses that the landlord is responsible for in the year the lease was written.
- Sometimes the owner will use the following year as the base year for the stop, and project the expenses for that period of time. Many times the term "expense stop" will not be used. Instead, the

landlord might choose to call the expense limit the “base year expenses”.

- This number is usually the total dollar amount of the landlord’s expenses for the year considered the base year. (Illustration 2-5)

Leasing Commissions

Many times, owners of multi-tenant properties will turn to leasing agents to secure leases for their building, including lease preparation, and sometimes, property management.

- The leasing agents make their money by charging the landlord a fee or commission for securing the tenant. That fee is usually based on the total revenue generated by the lease they secure. It is usually a small percentage somewhere in the 5% to 7% range.
- Leasing commissions may be paid up front, or may be paid out monthly/annually over the life of the lease. This is a real expense to many owners, and could be an appropriate operating expense deduction.
- Care should be taken when using this cost for mass appraisal to ensure that a common or consistent rate is used for all property types where it is applicable.
- If the commissions are not removed as an annualized expense, then they should be considered a capital expenditure and deducted as a lump sum cost from the final income estimate of value.
- It would be inappropriate to remove leasing commissions by either method if the capitalization rate you are using wasn’t developed in the same manner.

Tenant Improvements

As previously explained, tenant improvement costs are a capital expenditure that can be annualized and recovered by the landlord through the rent payment. Therefore, it can be considered an appropriate annualized expense deduction.

- The problem with using tenant improvements as expenses is the variety and extent of work done from one property to another. Some older properties aren't updated with new tenant improvements to attract new tenants. Others and new building owners might spend amounts from $5 a square foot upwards to $40 a square foot.
- As stated, this is an appropriate expense. However, it is very difficult to standardize this expense for mass appraisal use. Care should be taken when developing the expense models if this cost is to be included.
- If the tenant improvements recovery costs are included in the rent, an alternative to removing those costs on an annualized basis as a expense, would be to consider them as a one time capital expenditure and subtract that cost from the final income estimate of value. This amount would be the total TI costs necessary to bring non-current leased space up to market level rent as of the appraisal date.
- It would be inappropriate to expense out or subtract a lump sum cost for TI if the capitalization rate you are using wasn't developed in the same manner. This means that if the TI costs weren't removed from the income as an expense or subtracted from the adjusted sale price used in calculating the capitalization rate, then you cannot make that adjustment to your NOI or appraised value. (Illustration 2-6)

Deferred Maintenance

Deferred maintenance can be defined as the cost of items in need of immediate repair on the date of the appraisal.

1. The cost would be the cost to cure the problem.
2. Deferred maintenance can be a factor in the income approach to value. The appraiser can address the value two ways: “as is” or “as repaired”.

    a. The “as is” value would imply that the value reflects the problem needing repair, where the “as repaired” does not. For ad valorem tax, appraisers will appraise the “as is” value and consider problems the property is suffering from as of the date of the appraisal.

    b. One way to deal with the value lost due to deferred maintenance items would be to deduct the cost to cure the problem from the indication of value by the income approach. Since “as is” value is being sought, the appraiser is saying that they can only achieve market level rents, etc., if they account for the problem by deducting it from the total value.

    c. Another approach might be to reduce the rent rate and thereby reduce the value for the deferred maintenance items that way. The simplest approach and easiest to defend is to deduct the cost to cure.

    d. Special caution should be used when allowing or making deductions for deferred maintenance. Some owners call replacement allowance items deferred maintenance items. Examples of this would be parking lot or sidewalk repairs, roof repairs, HVAC replacements, and exterior painting.

    e. Unless the owner can show without a doubt, and/or the appraiser knows because of an inspection, using such a cost as a deferred maintenance deduction should not be allowed. The deduction should only be allowed if the NOI produced in the appraisal cannot be achieved without making the indicated repair.

Improper Expenses

Depreciation

- Depreciation will be considered in the income approach as recapture and handled as part of the capitalization rate.
- Depreciation found on operating statements for income tax purposes will generally not be the same as the recapture provision in the capitalization rate.

Debt service (principal and interest on mortgage)

- Principal and interest expense is a financing expense which is a part of the cost of capital necessary to acquire the property.
- Normally, a component for the cost of capital (the discount rate) is included in the capitalization rate.

Owner's personal expenses (income taxes, etc.)

Any business expenses not necessary for generation of income to the property are not valid operating expenses of the property. For example, income tax is an owner's personal expense.

Capital improvements

- Capital improvements are not items of annual expense. Typically, capital improvements will result in either increases in property value, economic life of the property, or income to the property.
- For example, the addition of a swimming pool to an apartment complex would not be considered a normal annual operating expense of the property. The swimming pool would be a capital improvement as the rent may increase because of this improvement and then there would be annual additional expense as a result of

the capital improvement. The swimming pool would also have an economic life assignment and thus not a proper annual expense.

Franchise fees and special corporation costs

A franchise is an agreement between a company and an independent owner in which the owner pays a fee to use the name, trademarks, and various other services offered by the chain. The franchise creates certain benefits and costs for both the owner and the chain.

Benefits to the Owner (Franchisee)

1. Identity, recognition and image (the image creates the price level economy, standard, or luxury and market, leisure, commercial, or convention).

2. Reservation or referral service (most have centralized reservation system that enables guest to reserve a room by using a toll-free telephone number; a good reservation system generates approximately 15-30% of a property‘s occupancy).

3. Chain advertising and sales (major franchises publish a directory in which each property is described and location and rate information is provided. Business generated through a central reservation system cannot support an individual lodging facility. Sales efforts on a local scale are also necessary).

4. Procedures manual (operating manuals are provided and each property is inspected manually).

5. Management assistance (provides specialized assistance in the development, planning, operations and marketing of a facility).

6. Group purchasing (chains require properties to purchase identity items such as towels, silverware, china, uniforms, etc).

Costs of the Owner

Required facilities (an affiliated property must have a minimum number of rooms and a food outlet on the premises or nearby. Membership fees (chains require an initial fee, which is determined by the size of the property; a continuing royalty fee, based on a percentage of room sales is paid each year; additional charges for advertising and reservations; franchise fees vary from 2% to 8% of gross rooms sales and terms range from 10 to 25 years. Required standards (must adhere to certain construction, design, operational and maintenance standards; failure to follow required standards can jeopardize the franchise).

Liability to the Owner

The chain offers no guarantee of financial commitment to the success of the property.

Benefit to the Chain (Franchisor)

Inexpensive, low-risk expansion (chains can expand operations with minimal capital and personal investment; cash flows are attractive to publicly held companies). Allied expansion (chains have developed allied businesses to support their franchises and company- owned operations such as interior designers, building contractors, furniture equipment, etc.)

Cost to the Chain Franchise

Service (chains must provide services described in the agreement). Quality control (inspection, supervision, and enforcement of franchise procedures and standards are essential).

Real Estate Taxes

Real Estate Taxes are a valid operating expense of property. However, because the ad valorem tax appraiser is appraising the property for the eventual purpose of determining the amount of ad valorem taxes, the appraiser must not make a deduction for real estate taxes. It is not possible to know the proper amount of real estate taxes until the correct amount of the assessed value is known. To make a deduction for real estate taxes, the appraiser would have to pre-determine the amount of value in order to compute the amount of real estate taxes. The ad valorem tax appraiser avoids this problem by making no deduction for real estate taxes from the income and instead the appraiser includes the effective tax rate as a component of the capitalization rate for the property.

- Real estate taxes are proper as an expense under certain conditions such as when appraising property for other than ad valorem tax purposes.
- Real estate taxes are not proper as an expense, under certain conditions, as when appraising property for ad valorem tax purposes.

PRIMARY RATES

<u>Overall Yield Rate (Y_o)</u> - The rate of return on total capital invested, including both equity and debt. The Y_o takes into consideration changes in net income over the investment period and net reversion at the end of the holding period; it

is applied to cash flow before debt service and is also called the Property Yield Rate.

Discount Rate - An interest rate used to convert future payments or receipts into present value. The discount rate may or may not be the same as the internal rate of return (IRR) or yield rate depending on how it is extracted from the market and/or used in the analysis.

- Safe rate - The base rate on the safest investments such as government insured investments.
- Risk rate - An amount in addition to the safe rate, which compensates the investor for the degree of risk in the investment.
- Rate for Non-liquidity - An amount in addition to the safe rate and the risk rate which compensates the investor for the time necessary to convert the real estate into cash.
- Rate for Management (Management of investment rate) - An amount in addition to the safe rate, risk rate, and non- liquidity rate which compensates the investor for the decision making process to manage the real estate investment.

Interest Rate - The price of money; the level of market interest carried by a debt instrument from the day it is created over the duration of its life. The rate of return or yield rate on debt capital, usually expressed as the nominal annual percentage of the amount loaned or borrowed. The rate of return on investment irrespective of, and independent of, any capital recovery received or demanded by the investor; specifically, the rate on borrowed money.

Overall Capitalization Rate (R_0) - An income rate for a total real property interest that reflects the relationship between a single year's net operating income expectancy and the total property price or value; used to convert net operating income into an indication of overall property value.

Property Yield - The dollar return on the entire real property from all sources, i.e., the annual net operating income including any gain or loss in the original property investment at termination.

Yield Rate (Y) - A rate of return on capital, usually expressed as a compound annual percentage rate. A yield rate considers all expected property benefits, including the proceeds from sale at the termination of the investment.

Yield to Maturity - In finance, the total rate of return that would be realized on an investment such as a bond if purchased at the current market price, held as an investment, and redeemed for the principal amount at maturity.

Yield Capitalization - The capitalization method used to convert future benefits into present value by discounting each future benefit at an appropriate yield rate or by developing an overall rate that explicitly reflects the investments income pattern, value change, and yield rate.

Equity Yield Rate (Y_E) - A rate of return on equity capital as distinguished from the rate of return on debt capital; the equity investor's internal rate of return. The equity yield rate considers the effect of debt financing on the cash flow to the equity investor.

- A true annualized rate of return on equity capital including (in addition to the average annual dividend) the full effect of any gain or loss from resale at the termination of the investment; a single discount rate that will make the present worth of all the future equity benefits equal the equity investment.

Recapture rate - Provides for the return of the investment in the wasting portion of the asset. This is the same as the depreciation rate for the improvement. Recapture takes into consideration the remaining economic life of the improvement.

Effective tax rate (ETR) - Reflects the relationship between the real estate taxes and the value of the property. The tax rate expressed as a percentage of market value. ETR can be calculated in two methods.

- Effective tax rate = Assessment ratio x Mill rate (nominal tax rate)
- Effective tax rate can also be calculated by dividing the taxes by the market value. Illustration 2-7

DIRECT CAPITALIZATION

Direct capitalization is typically used in mass appraisal in that a net operating income is calculated and then divided by a capitalization rate to determine market value.

The capitalization rate is composed of the discount, recapture and effective tax rates. Within the direct capitalization process, a determination must be made on how to handle taxes when using sales to extract the rate. It would be typical that the actual taxes be subtracted as an expense as the purchaser or investor in the property would consider that as an expense in setting the sale price. If the taxes are subtracted as an expense, then the capitalization rate would consist of the discount and recapture for improved properties and would only be the discount rate for vacant land. A capitalization rate derived directly from the market does not distinguish between discount and recapture rates. (Illustration 2-8)

Once the rate is determined through direct capitalization, then the actual effective tax rate is added back to make a "loaded rate." Example: Loaded to allow for the proper deduction of property taxes through the process of capitalization.

Replacement Reserves must be considered with the rate development the same as taxes. If reserves are allowed or included in the expenses, then the income and the rate are lower. If the reserves are not an expense item, then the income and rate would be higher. If done consistently, the values should come out the same. Illustration 2-9

GENERIC CAPITALIZATION FORMULAS

The lRV formula (equation)

I (income) = Rate x Value

R (rate) = Income / Value

V (value) = lncome / Rate

Illustration 2-10

The VIM formula (equation) - the VIM equation is also known as the gross income multiplier.

V (value) = Income x Multiplier

I (Income) = Value / Multiplier

M (multiplier) = Value / lncome

Illustration 2-11

Use of income / rent multipliers

- Multiplier is another word for Factor.
- Works with the VlM (sometimes called VIF) formula.
- This formula has more limited application and often is used as a check against the normally used capitalization rate in the income approach method. Both are "income methods."
- Use of the PGIM
  a. A PGIM is calculated by sale price / gross income for the property. b. Once calculated for several similar assets, a PGIM may be multiplied against the income of a property to obtain an estimate of value.
- ln order to have confidence in the PGIM, you need to have the following: a. Several sales with the known gross income. b. Sales should be for similar properties having the same comparability factors as that for capitalization rates. 1) Close remaining economic lives. 2) Similar operating expense ratios. 3) Similar land-to-building ratios.
- Location should be similar or adjustments can be made based upon determined external obsolescence factors. Illustration 2-12 & 2-13
- The multiplier can also be developed for the Effective Gross or the Net Operating Income. Those formulas are shown below.
  a. Effective Gross Income Multiplier (EGIM) = Sale Price / Effective Gross

b. Net Operating Income Multiplier (NOIM) = Sale Price / Net income

c. Use of the NOIM may be very limited. In order to develop and apply this multiplier, there is an assumption that all the comparables used to develop the multiplier and the subject are very similar including the Operating Expense Ratio (OER).

ILLUSTRATION 1-1

OVERAGE RENT

A typical overage rent is calculated at the same percentage level as the base rent. A tenant leases 1,000 square feet at a shopping center. The tenant's base rent is $25.00 per square foot and 10% of the tenant gross sales exceeding $250.00 PSF. The tenant will pay 10% of this amount to the owner as an overage rent. Over time, the tenant has gross sales of $350.00 PSF. Therefore, the tenant will pay $25.00 PSF in minimum rent and $10.00 PSF in overage rent (10.0% X ($350.00 - $250.00)), resulting in a total rent of $35.00 PSF. lf the tenant falls below its projected gross sales amount, it still must pay the minimum rent ($25.00 PSF). Therefore, percentage leases involve no downside risk to the owner, only increased profits. Also, overage rent is a function of the owner's business expertise and the tenant's business expertise.

ILLUSTRATION 1-2

DETERMINING RENT TYPES

1. Is it possible for the value of a leased fee estate to exceed the market value of the real estate on an un-leased basis? (An un-leased basis is a property available for lease or one that is under lease without a formal document). Yes, the excess value might indicate 'negative leasehold' or an intangible value of a lease premium, which is over and above the value of the realty.

2. An excess of contract rent over market rent indicates an advantage to the lessee. True or False?

False

EXERCISE 1-1

DETERMINING RENT AMOUNTS

Lease Valuation: Lease Term 10 years Contract Rent $800 per month plus 6% gross sales Actual Rent Gross sales range from $15,000 to $30,000 per month, averaging $20,000 per month Market Rent $1,000 per month plus 5% gross sales

1. What is the contract rent?

2. What is the range in actual rent?

3. What is the average actual rent?

4. What is the market rent?

5. What is the overage rent, based upon the lease if the actual sales in one month were $18,000?

EXERCISE 1-2

DETERMINING LEASEABLE AREA & RENT

Subject property is a three-story building with dimensions of 60 x 120. The first floor has a common area of 400 SF and each floor has stairwells, elevators and hallways of 800 SF. If the rent is on a Net Leaseable Area basis and the first floor rents for $18 per SF and the other floors at $16 per SF, what is the PGI?

ILLUSTRATION 1-3

RENT CONCESSIONS

METHOD 1:

Take a percentage of the total income from the potential gross income (PGI).

As an example: 5% of the revenue was lost due to concessions.

Multiply 5% times the PGI to get that loss in revenue.

PGI = $34,800 x 0.05 = $1,740 rent concession

METHOD 2:

Reduce the rent rate to an effective rent rate considering the reduced or free rent or higher than market TI allowance.

The tenant was given 3 months free rent on a 5 year (60 month) lease. The full service rent in the lease is $20.00 per SF. You can calculate effective rent by dividing 3 by 60 to get 5% and then deduct 5% from the original rental rate of $20 to get an effective rate of $19 for the life of the lease.

3 / 60 = 0.05 Percent of lease than was given as a concession.

0.05 x $20 per SF = $1.00

$20.00 - $1.00 = $19.00 Effective rent considering the concession.

If the property had 9,000 SF, the rent concession would be:

9,000 x $1 x 5 years = $45,000.

ILLUSTRATION 1-4

RENT CONCESSIONS

Using the information for Method 2 on Illustration 1-3, as shown below, what would be the rent loss if the rent for Years 4 & 5 increased to $22 per SF?

The tenant was given 3 months free rent on a 5 year (60 month) lease. The full service rent in the lease is $20.00 per SF. You can calculate effective rent by dividing 3 by 60 to get 5% and then deduct 5% from the original rental rate of $20 to get an effective rate of $19 for the first three years of the lease. The effective rent for Years 4 & 5 would be $20.90 or a rent loss of $1 .10 per SF.

As this is a 5-year lease, it may be appropriate to discount the rent concession over the life of the lease. For this example, the rate used for discounting will be 9%.

Solving with
Compound
Interest Tables

See the annual tables in the Appendix - PV of $1 and PV of $1 per period				
$9,000	x	2.531295	=	$22,781.6550
$9,900	x	0.708425	=	$7,013.4075
$9,900	x	0.649931	=	$6,434.3169
		PV of Rent Loss		$36,229.3794

EXERCISE 1-3

RENT CONCESSIONS

1. A property owner has given a tenant 6 months of free rent on a 6-year lease. If this is a full service lease at $24 per SF and is for 4,200 SF, what is the rent concession not considering the net present value of the rent loss?

2. What would be the rent concession for the above property if there were step-ups in the lease as listed below and a discount rate of 9%? Years 3 & 4 rent to be $25 per SF Years 5 & 6 rent to be $26.50 per SF?

ILLUSTRATION 1-5

PASS-THROUGH EXPENSES

An office building was recently remodeled and has signed new leases with the tenants. The new leases state that all expenses above the base year annual expenses of $365,000 will be pass-through to the tenants based upon the net leaseable area per tenant. Annual expenses for the next year were $392,000. If tenant A has 12,000 SF of the total NLA of 60,000 SF, what would be the amount of pass-through expense to tenant A and what is the expense stop per SF?

$392,000 - $365,000 = $27,000 of pass-through expenses

12,000 / 60,000 = 0.20 20% of the space for tenant A

$27,000 x 0.20 = $5,400 Pass-through expense for tenant A

$365,000 / 60,000 = $6.08 per SF expense stop

EXERCISE 1-4

PASS-THROUGH EXPENSES

Base year expenses are $651,160 and current year expenses are $702,543. You are leasing 4,000 SF of the total NLA of 85,000. What is the amount of your pass-through expenses and what is the expense stop per SF?

ILLUSTRATION 1-6

TENANT IMPROVEMENTS (TI)

Subject has 10,000 SF of NLA and was renting for $18.25 per SF. lt is anticipated that the TI costs will be $185,450 in order to attract a new tenant for a 5-year lease. What would the rent be after making the tenant improvements?

EXERCISE 1-5

TENANT IMPROVEMENTS (TI)

In order to fill the vacant space, a property owner will have to make TI of $426,500 to the NLA of 14,200 SF. The previous tenant was paying $23.00 per SF and was considered market. If the new lease will be for 6 years, what will be the necessary rent to recover the cost of the TI?

ILLUSTRATION 1-7

Computing an Effective Tax Rate

Formula:

Effective Tax Rate = Assessment Ratio x Tax Rate

Example:

Assessment Ratio of 40%

Current Tax Rate is based on 30 mills (30/1,000)

Step 1: Divide 30 by 1,000

30 / 1,000 = 0.030

Step 2: Multiple assessment by the result of Step 1

Effective Tax equals: **0.40 x 0.030 = 0.012**

ILLUSTRATION 1-8

CAP RATE WITH TAXES AS AN EXPENSE

Office Building Parcel #: 003-033

Sale Date: Current Year		Sale Amount: $236,140
Actual Gross Rent: $37,484		Square Feet: 4,436
Potential Gross Income		$ 37,434 PGIM = 6.3
Vacancy & Collection Loss (5%)	-	$ 1,874
Effective Gross Income		$ 35,610
Management (5%)	-	$ 1,781
Expenses (actual)	-	$ 2,201
Net Income Before Taxes		$ 31,623
Property Taxes for previous year	-	$ 9,022
Net Income		$ 22,606
Sale Price	/	$ 236,140
Capitalization Rate		0.0957 = 9.6%

ILLUSTRATION 1-9

CAP RATE & REPLACEMENT RESERVES

Example **with** reserves as an expense:

Effective Gross Income	$ 40,000
Operating Expenses	- 10,000
Reserves	- 2,000
Net Operating Income	$ 28,000
Sale Price	$ 300,000
Indicated Overall Rate	0.0933

Example **without** reserves as an expense:

Effective Gross Income	$ 40,000
Operating Expenses	- 10,000
Net Operating Income	$ 30,000
Sale Price	$ 300,000
Indicated Overall Rate	0.1000

Note that when reserves are included as an expense that the overall rate is less. That states that an investor is paying the same price but the return would change as the net income changes.

ILLUSTRATION 1-10

IRV

A corner lot in a business district is used as a parking lot. The monthly income earned by the property is $12,500. The owner's required rate of return for this type of property is 10% per year. What is the value of this property?

$$\frac{\text{NOI}}{\text{Cap Rate}} = \text{Value}$$

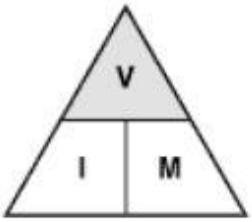
Income = $12,500 x 12 = $150,000 / 0.10

V = $1,500,000

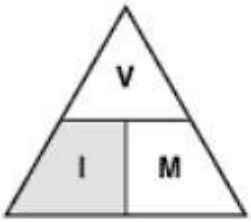
ILLUSTRATION 1-11

VIM

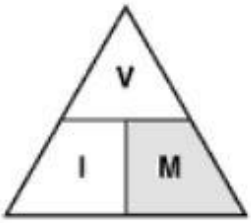
You derived the value of $1,500,000 for the corner parking lot in a business district on the prior page. You calculated the annual income earned of $150,000. Using VIM, calculate the Gross Income Multiplier for this property. The VIM formula may be used to calculate a multiplier at any income level; Potential Gross (PGIM or GIM), Effective Gross (EGIM), or Net (NIM).



V = I × M



I = V ÷ M



M = V ÷ I

$1,500,000 divided by $150,000 = 10 (Multiplier)

ILLUSTRATION 1-12

PGIM

Sale Price = $ 227,000

Gross Income = $ 42,200

What is the PGIM? $

227,000 / $ 42,200 = 5.38 or 5.4.

If a comparable property has a Gross Income of $56,500, the indicated value by using PGIM from above would be:

5.4 x $56,500 = $305,100.

ILLUSTRATION 1-13

EGIM

An EGIM for a restaurant was determined to be 5.8. The subject you are appraising is very similar to the sold property but is in a less desirable location. Your analysis has shown that the sale property is in a location where there is a 10% reduction (90% good economic factor) and the subject has a 25% reduction (75% good economic factor). The EGIM would be adjusted as follows:

0.75 / 0.90 = 0.83

0.83 x 5.8 (EGIM) = 4.8 EGIM for the subject

If the subject has a gross income of $12,000, then the value indication would be: $57,600 ($12,000 x 4.8)

The EGIM would be lower as the property would be less valuable.

REVIEW QUESTIONS

1. Annual economic rent for the property at 100 percent occupancy is _____ _____ _____.

2. The rental income that a property would most probably command in the open market is _____ _____ .

3. The fixed portion of the rent under the terms of a percentage lease is _____ _____ _____ .

4. The percentage rent paid over and above the guaranteed minimum rate is _____ _____ .

5. The amount of which contract exceeds market rent at the time of the appraisal is _____ _____ .

6. The area within a building or structure that is actually occupied by an individual tenant is _____ _____ _____ .

7. A loss in income for the owner because of an agreement with the tenant to either reduce the rent on short term basis, or allow free rent for a short term, or supply higher than normal tenant improvement (Tl) work is called a _____ _____ .

8. A property owner has given a tenant 6 months of free rent on a 5-year lease. If this is a full service lease at $20 per SF and is for 6,000 SF, what is the rent concession not considering the net present value of the rent loss? _____

9. _____ is a term used when a property has a vacancy rate greater than the market vacancy rate as of the date of the appraisal.

10. The tenant may have to pay CAM expenses. What does CAM mean?
_____ _____ _____

11. Base year expenses are $415,650 and current year expenses are $522,755. You are leasing 5,000 SF of the total NLA of 65,000. What is the amount of your pass-through expenses? _____

12. An OAR for an improved property that used real estate taxes as an expense, would be composed of _____ and _____ rates?

13. A property has an EGI of $65,200 with operating expenses of 30% plus real estate taxes of $7,200. What is the indicated OAR with taxes as an expense if the property sold for $418,000? _____

14. Using the answer to number 13, if the assessment rate is 40% and the mill rate is 30 mills, what is the loaded rate? _____

15. _____ _____ costs may be necessary to rent space to a new tenant and are a capital expenditure that can be annualized and recovered by the landlord through the rent payment.

16. Subject has 14,000 SF of NLA and was renting for $20.25 per SF. It is anticipated that the TI costs with be $235,450 in order to attract a new tenant for a 5-year lease. What would the rent be after making the tenant improvements? _____

17. The formula for calculating PGIM is _____
_____ + _____ _____
_____ .

18. Subject property has two floors each having gross area of 8,000 SF. Floor one has a total of 900 SF of common area and floor two has 400 SF of common area. Rent for floor one is $12.00 SF of NRA and floor two is $13.00 per SF of NRA. What is the PGI? _____

19. Valuing a property "as is" or deducting for the cost to cure are two methods of recognizing _____ _____ .

20. If reserves are included in the expenses, the overall rate would be _____ .

DIRECT CAPITALIZATION v. DISCOUNTED CASH FLOW

To estimate value with direct capitalization, a property's expected net operating income (NOI) is divided by the overall capitalization rate. (See Fig. 1-1) Estimating value with DCF analysis requires estimates of each year's NOI along with the property's expected reversion value at the end of the holding period. The appraiser normally uses income capitalization to estimate the reversion. (See Fig. 1-1) These expected cash benefits are then discounted at the appropriate rate to obtain the market value estimate.

Although these calculations are simple and straightforward, they depend on the appraiser's assumptions or estimates. When using direct capitalization, the property's stabilized NOI must be estimated. This estimate is developed from market data (rental rates, vacancy and collection loss rates and operating expense data) for comparable properties in the market area; it represents the appraiser's estimation of how the property should perform.

Because the appraiser's opinion is based on observed market data, it's difficult to object to the NOI estimate. When a market is "normal," the idea of stabilized NOI is particularly useful.

However, there are two areas of concern. One: what if the property has significant vacancy at the time of the appraisal? Clearly, no one develops a property with the expectation of significant, permanent vacancy. So, the appraiser may use a market vacancy rate (or the anticipated vacancy rate when the property is fully leased) rather than the property's actual vacancy rate. Two: if the property's future NOI is expected to increase because of greater demand for space that leads to higher rental rates, direct capitalization of a single year's NOI may understate the property's value.

Because DCF analysis permits annual adjustments in rental rates, vacancy and collection loss rates and operating expenses, DCF analysis can be used to reflect a buyer's expectations of increasing NOI over time. When a property is expected to become fully leased during the next three to five years, for example, the current vacancy rate can be reduced until the desired occupancy is reached.

Accurate forecasting of expected changes results in realistic NOI estimates throughout the holding period—a result far superior to capitalizing a single year's NOI.

DCF analysis is ideally suited for situations such as this. However, DCF analysis does not add much useful information when the subject property is fully leased and no changes in occupancy or the external environment are anticipated that will affect NOI. As shown in Figure 1-1, the present value of a series of equal annual cash flows is the same as the capitalized value.

It is not an error to use a DCF analysis to value a property when no significant change in NOI is expected. However, it should be understood that this technique's results are no better than those produced by a correct application of direct capitalization. Of course, if both methods, properly used, result in similar values, the estimated value has greater credibility.

Figure 1-1 Comparison – Direct Cap and DCF

Figure 1-1. Estimating Value with Direct Capitalization and DCF Analysis

Direct Cap:

$$\text{Value} = \frac{\text{NOI}}{\text{Capitalization rate}} = \frac{\$10{,}000}{.06} = \$166{,}667$$

DCF (Yield Cap)

Year	Income	Reversion	PVIF	PV of CF
1	$10,000		0.943396	$9,434
2	$10,000		0.889996	$8,900
3	$10,000		0.839619	$8,396
4	$10,000		0.792094	$7,921
5	$10,000	$166,667	0.747258	$132,016
Present value of Cash Flows at 6%				$166,667

DISCOUNTED CASH FLOW

From a quantitative viewpoint, investing in real estate is to some extent like investing in stocks. In order to profit in real estate investments, investors must determine the value of the properties they buy and make educated estimates about how much profit these investments will generate, whether through property appreciation, rental income or a combination of both.

COMPOUND INTEREST TABLES

Knowing the six functions of a dollar, three that compound and three that discount, is necessary for the discussion of discounted cash flow in this section.

Compound interest expresses the time value of money — a dollar received today is worth more than a dollar to be received in the future. There are six interest functions and thus six tables, used to measure present worth and future worth. The terms worth and value are inter-changeable.

> Typically, a single deposit or payment is assumed to be made at the beginning of the period where periodic deposits are assumed to be made at the end of the period.

Effective interest rate vs. nominal interest rate

1. The effective interest rate is the nominal interest rate divided by the number of conversion periods in one year. For example, if the nominal interest rate is 9% (0.09), the effective interest rate for monthly compounding would be 0.0075 (0.09/ 12).

2. The effective interest rate is used in all calculations for compounding.

Future worth of $1

1. Origin of all present worth and future worth functions.

2. Calculates the growth of a single deposit over a specific time period at a specified rate of interest.

3. Also known as the compound amount of $1.

4. The reciprocal of the present worth of $1.

Future worth of $1 per period

1. Calculates the growth at compound interest of a series of deposits.

2. Also knows as the compound amount of $1 per period.

3. The reciprocal of the sinking fund factor. Practical 2-1 through 2-4

Sinking fund factor

1. Calculates the amount of periodic payment necessary to accumulate to a specific sum.

2. Reciprocal of the future worth of $1 per period. Practical 2-5 & 2-6

Present worth of $1

1. Calculates the present worth of a single future payment.

2. Reciprocal of the future worth of $1.

3. Also known as the reversion factor.

Present worth of $1 per period “

1. Calculates the present worth of a series of future level income payments (level annuity).

2. Also known as the annuity factor or the lnwood coefficient.

3. Reciprocal of the partial payment factor. Illustration 2-7 through 2-10

Partial payment factor

1. Calculates the periodic payment necessary to amortize a loan at a specific interest rate over a specific period of time.

2. Also known as the installment to amortize $1. This provides the lender the return on and of the borrowed funds.

3. Reciprocal of the present worth of $1 per period.

4. Sum of the effective interest rate and sinking fund factor. Illustration 2-11 through 2-12

5. Mortgage constant is the annual total debt service, including payments of interest and amortization, expressed as a percentage of the amount of the loan

Formula is total annual payments / amount of loan Illustration 2-13

YIELD CAPITALIZATION

Overall Yield Rate (Y_o): The rate of return on total capital invested, including both equity and debt. The Y_o takes into consideration changes in net income over the investment period and net reversion at the end of the holding period; it is applied to cash flow before debt service. It is also called the Property Yield Rate.

1. The income over the holding period is a series of incomes.

2. Reversion provides for recapture when using discounting but not in direct capitalization.

Equity Yield Rate (Y_E): A rate of return on equity capital as distinguished from the rate of return on debt capital; the equity is the investor's internal rate of return. The equity yield rate considers the effect of debt financing on the cash flow to the equity investor.

1. A true annualized rate of return on equity capital including (in addition to the average annual dividend) the full effect of any gain or loss from resale at the termination of the investment; a single discount rate that will make the present worth of all the future equity benefits equal the equity investment.

Equity dividend rate — Uses before-tax cash flow and is often referred to as cash-on-cash return.

Calculated by (net operating income - debt service) / equity investment.

Equity investment is the amount of money that the investor must contribute to purchase a property.

The equity dividend rate is also called the equity capitalization rate.

Internal Rate of Return (IRR) - The annualized yield rate on capital that is (or could be) generated by an asset or within a group of capital assets over a period of ownership.

> Internal Rate of Return is the rate that discounts all future cash flows to a net present worth equal to the original investment.

DISCOUNTED CASH FLOW (DCF)

1. A yield capitalization method used to calculate the present value of anticipated future cash flows.
2. Isolates differences in the timing of cash flows. Net cash flows from all time intervals, in the analysis are discounted to present value by an appropriate discount rate.
3. Using, data similar in investment holding period, net income estimates and financial terms, DCF should produce estimates similar to direct capitalization.

Yield capitalization

1. Yield capitalization is used with DCF, another method for converting income into value.
2. Yield methods require certain assumptions that vary. The assumptions include estimating such factors as the required rate of return on investment, the remaining economic life of the property, an investment holding period, and the income path. Anticipated depreciation or appreciation are reflected in the reversionary value.
3. It is a year-by-year breakdown of future benefits (net operating income) an investor expects to receive, discounting each year of the future benefit to a present value as of the appraisal date.

4. The net operating income that will be used for the annual discounting is normally after the deduction of property taxes. However, as the ad valorem process involves estimating value for tax purposes, this would almost appear to be accepting that the taxes were correct and thus the previous value must also be correct. DCF is looking at value through an investor's viewpoint and property taxes are an investment consideration. A method to handle taxes will be shown later.
5. One time that real estate taxes may be shown as an expense would be when the taxes are an expense stop for the owner of the property. At that point, the taxes are a fixed expense. This can occur normally in a lease but is sometimes seen in new leases just prior to or immediately after a reappraisal where property values have not been updated in a long period of time.
6. The process also converts the reversionary value into a present value. Reversion is the right of possession commencing on the termination of a particular estate. A more simple definition is when you sell the property at the end of the holding period and your estate (ownership) is no longer the property but you have reverted, or converted, the asset back to cash. Illustration 2-16

Investment analysis

1. DCF can also be described as a valuation method used to estimate the attractiveness of an investment.
2. This is a tool and not a guarantee. Small changes in inputs such as rent, expenses, capitalization rate and reversion can result in large value changes.
3. Meaningful valuations depend on the users' ability to make solid cash flow projections. A single, unexpected event can immediately make a DCF model obsolete.
4. There are properties that are truly investment type properties in which yield capitalization and the application of DCF should be considered.
5. Often we hear comments about investment grade properties. These are generally good quality structures that are rented on short-term leases

such as 5-10 years, have renewal options and are newer improvements. Examples of these would be:

a. Office buildings
b. Strip centers
c. Shopping malls
d. Apartment buildings could also be valued by DCF, although the leases are commonly month-to-month or on an annual basis.
e. Office buildings leased to a government agency.

Other potential applications of DCF may be property that is:

a. Currently in a rent up period, such as a new property.
b. Property that is suffering from high vacancy with an expected increase in demand for the space.
c. An older property suffering from high vacancy under-going remodeling and updating to meet an anticipated need.
d. Contaminated properties with multiple-year clean-up costs.

DCF also lends itself to application on some unique properties such as mineral quarries, mines, sanitary landfills and land development.

Older properties that are being leased on a long term could also be candidates for the application of DCF. However, buildings that are nearing the end of their economic life would not be a candidate for using DCF. The longer the DCF projection, the less reliable the answer generally will be.

A typical investor is going to analyze the past, current and future performance of a property prior to investing. In order to measure the response of the investor, DCF should be considered.

Investors analyzing the performance are similar to stocks or mutual funds. Analyses give you the last quarter, year-to-date, one- year, 5-year and life of the fund performance to help you determine if it is a good investment.

DCF permits annual adjustments to rent, vacancy and expenses to reflect a buyer‘s expectations of increasing or decreasing net

operating income over time. When a property is going to reduce the vacancy over a few years, the current vacancy rate can be reduced until the desired occupancy can be reached. ln direct capitalization, we use one year based upon historical data. But the income approach definition talks about anticipated future benefits.

A weakness of DCF is the reliance upon forecasting what is going to happen to the income stream, expenses and what the value of the property will be at the end of the holding period. Some of the considerations in making the forecasts are:

1. Current rent versus market rent and rent adjustments within the contract. This is much easier if we have the subject rent to compare to others in the determination if the rent is at market level.
2. Renewal options with any escalation provisions.
3. Anticipated tenant turnover. There is bound to be some turnover unless the contract coincides with the holding period. You may need to anticipate some vacancy after the conclusion of a contract to allow for tenant improvements prior to a new tenant.
4. Operating expenses that are the responsibility of the owner and consideration of any pass-through expenses to the tenant.
5. Changes in the Consumer Price Index that may affect the rent or expenses.
6. Residual value of the property at the end of the holding period and any associated selling or transaction costs.

Application of DCF can be summarized as follows:

1. Select a holding period for the investment. Normally the holding period will be for 5, 10 or 15 years with 10 being the most typically used.
2. Forecast all future gross and net cash flows.
3. Select an appropriate yield rate.
4. Convert the future benefits of each year’s net cash flow and the reversion to a present value.

Quantity, variability, timing and duration of the cash flows

1. The quantity of the rent is simply how much is to be received.
2. Variability is how much change or dispersion there is in the rent from year to year.
   a. Rent may be received as an annuity.
   b. Annuity is the right to receive money or its equivalent in (usually) fixed equal amounts and at regular intervals for a definite or indefinite term.
   c. An annuity can be level, increasing or decreasing and does not have to be annual, so long as the amounts are scheduled or can be forecast.
   d. If an annuity in not level, it is referred to as a variable annuity.
   e. If the annuity (rent) is decreasing, the property may be nearing the end of its economic life. Once a property reaches the end of its economic life, it no longer contributes value and generally is not a candidate for DCF.
   f. If the net operating income is the same each year then this is referred to as a constant or level terminal annuity. lt is a terminal annuity because the income stream is going to stop at the end of the holding period.
   g. Real estate rent or income would be considered an annuity.

Timing of the income stream is when the rent will be received, whether it is all at the beginning of the year, monthly or at year end.

Income received at the end of each period is referred to as an ordinary annuity.

Typically the rent is received monthly and the income is recognized within DCF as occurring at year end.

A lease that calls for the payments to be made at the beginning of each month creates an annuity in advance.

Duration of the income stream is how long it will last. That is until the end of the holding period.

Estimation of PGI Potential Gross Income (PGI) must be calculated for the subject property. As DCF uses several years, typically 5-10 as the basis for determining value, income must be recognized for each year.

Historical data for a property can also help to develop new rents by recognizing rent increases.

> Normally a lease will set a rent for the base year and either state the rent amount for each year of the lease or it will state something to the effect of annual rent increases by 3% compounded annually.
>
> In order to develop a DCF for a property, a percentage of change should be recognized within the calculation that is shown in the lease or is reasonable in relationship to similar properties.
>
> In mass appraisal, leases are not often provided or accessible. The building of an in- house income data bank may help in determining typical rent increases in the market place plus typical escalation periods. Illustration 2-17

Vacancy & collection loss

1. The percentage reduction from PGI is the same process as with direct capitalization except you again are projecting into the future.
2. If within the holding period the current lease expires, additional consideration may need to be given for vacancy due to improvements necessary to attract the next tenant.

Miscellaneous income

1. Calculated in the same manner as for direct capitalization, but done for each income projection period.

Expenses

1. Expenses can be operating or reserves or both.
2. Expenses are calculated the same as in direct capitalization, but done for each income projection period.

3. Just like income, expenses have to be projected into the future. Historical expense trends can be measured from the subject's actual expenses, based upon lease information from the subject's lease or from market data of comparable properties.
4. Expenses normally increase annually and a typical investor will recognize that. Therefore, if the investor were paying part or all the expenses, an annual increase would be appropriate.
5. If an expense stop is involved, then the expenses may only increase for a limited time or not at all based upon the terms and timing of the lease versus the appraisal date.
6. The appraiser must be aware also if some or all of the expenses are a pass-through to the tenant and thus not a deductible expense.

Selection of a capitalization rate

Direct capitalization is typically used in mass appraisal in that a net operating income is calculated and then divided by a capitalization rate to determine market value.

Under direct capitalization, you must determine how reserves are accounted for, either as an expense or part of the rate.

A capitalization rate can also be determined by the mortgage equity method.

1. Mortgage-Equity Analysis is a technique used to estimate the value of a property from knowledge of the equity yield rate, typical terms (including the interest rate, the loan-to- value ratio, the term of the loan, and the amortization provisions), the holding period, and the percentage by which the property will appreciate or depreciate over the holding period.
2. Mortgage Constant: Annual debt service expressed as a percentage of the initial principal amount of the loan.

The yield rate is the return on investment applicable to a series of incomes that results in the present worth of each.

1. A yield rate to be applied in DCF should be extracted from the market as this reflects the actual activity and return on investment.
2. One method of extracting this is through the lnternal Rate of Return (IRR).
   a. Internal rate of return is the annualized yield rate on capital that is (or could be) generated by an asset or within a group of capital assets over a period of ownership.
   b. Internal Rate of Return is the rate that discounts all future cash flows to a net present worth equal to the original investment.
   c. The internal rate of return is calculated, usually by trial and error, from knowledge of the relevant cash flows.
   d. The IRR is most accurately calculated after the reversion. Appraisers need to use diligence in forecasting future value. This will enhance the credibility of the value estimate.
   e. An example of calculating the internal rate of return using the HP12C is shown in Illustration 2-18 & 2-19

Reversionary rate

1. It is generally recognized that the rate to calculate the reversion may be different (higher) than the one used for the income stream.
2. The reversion occurs at the end of the holding period, this requires a longer projection period than most of the income stream, a higher degree of risk and a reduced remaining economic life of the improvement.
3. The income stream is often easier to project, especially with a long-term contract, than what the selling price of the subject property would be say 10 years from the date of the appraisal.
4. The rate used to calculate the reversionary value is sometimes referred to as the terminal rate (R_n).

Reversion

As stated earlier, reversion is the right of possession commencing on the termination of a particular estate.

Any conversion of income or reversion back to a present value is called discounting.

The reversion represents the return if the capital investment that was made in the property.

The recapture of the investment may be received in the income stream and the reversion or it may totally be received in the reversion.

If the capitalization rate on a purchased property is really low, the investor may be stating they are looking to cover cost and return on the appreciation of the property at reversion.

If the investment is not totally recaptured between the income stream and the reversion, then the yield rate is actually negative.

A higher rate would be used if the net present value of the reversion is greater than the net present value of the income stream.

This would seem logical as the investor is waiting a longer period of time for the recapture and thus taking greater risk.

Another method requires the appraiser to estimate if the original investment will stay the same, increase in value or decrease in value when estimating the reversionary value.

This would require a market analysis of supply and demand projections. Historical data reflecting trends within the market, based upon sales, would be a good starting point.

Just as in the rate calculation, this historical perspective should be considered but ultimately the appraiser is forecasting a future event, often 10 years in the future.

Typically there would be some expenses associated with the reversion if the investor is going to sell at the end of the investment period.

1. These expenses may be items such as brokerage fees, commissions and legal fees.

2. These expenses should be subtracted from the reversion prior to discounting to present value.

Discounting

1. Discounted cash flow analysis relies on compound interest measurements to convert future dollars into their present value equivalents.
2. Because dollars to be received in the future are worth less than current dollars, successive cash inflows and outflows are discounted at a selected rate to their present value as of a given date.
3. The sum of the positive and negative present values is the indicated value by use of DCF. Income could be negative for some years.
4. If the rent received is a level ordinary annuity, the calculation for the present value is relatively easy by using the Present Worth of 1/Period column. Illustration 2-20 & 2-21

PROFITABILITY INDEX

A profitability index measures the relationship between the projected future benefits of a project or investment and the initial capital outlay and is an additional tool for measuring the quality of a real property investment.

The formula for calculating the profitability index is:

Present value of anticipated future benefits / initial investment

Present value of the anticipated future benefits would be for the cash flows and the reversion.

A ratio of 1 implies that the return equals the desired yield rate. The higher the ratio, the more attractive the investment is.

Example: Using the information from Illustration 2-18 and the NPV, and including the original cash outflow, is $106,827. The profitability index would then be:

$106,827 / $90,000 (initial cash outflow) = 1.187

BREAKEVEN POINT

Another investment analysis tool, the breakeven point, is the point where total income equals total expenses including debt service. According to Real Estate Appraisal Terminology published by the American Institute of Real Estate Appraisers, it is, “the intersection of the total cost line and the total revenue line.

This analysis will assist in measuring the length of time before an investment will become profitable. It can be beneficial in projects such as new construction and for land being converted into subdivisions.

It is also referred to as a breakeven ratio where the ratio is:

Effective gross income / (Expenses + Annual Debt Service)

Example of the breakeven point:

Annual operating expenses for an apartment complex are estimated at $200,000 per year. Debt service requirements are $250,000 per year. Therefore, the total cash requirements are $450,000. Then, if all apartments are rented all year, the effective gross income would be $600,000. The breakeven point is 75% occupancy ($450,000 / $600,000) because at that point, effective gross income would equal cash requirements.

PAYBACK PERIOD

The payback period is the length of time required for net income cash flows to equal the original cash outlay. The faster the recovery of the outgoing cash flow the better.

Cost + Annual Cash Inflow is the formula.

This formula does not consider the time value of money and also ignores benefits or cash flows after the initial cost is recovered.

ABSORPTION OR BELOW MARKET OCCUPANCY LEVELS

1. Absorption is a term used when a property has a vacancy rate greater than the market vacancy rate as of the date of the appraisal.
2. Absorption or below market occupancy would normally occur in A new construction, such as offices, retail, and multi-family improvements and where obtaining a normal occupancy derived from market analysis will occur over time through absorption. Furthermore, this vacancy problem would need to be considered a temporary problem to be classified as absorption.
3. A long-term problem that cannot be corrected at reasonable cost is really a basic vacancy problem and should be treated that way. If the absorption procedure is used, this would be an adjustment to the income approach, and possibly the sales comparison approach, as well as the cost approach.
4. A fee simple purist may argue that if a fee simple value is the objective, only market factors and market rates can be used in estimating value. Those that subscribe to this idea will not adjust an income approach to value if a property is suffering from below market occupancy as of the date of appraisal.
5. However, some jurisdictions will adjust for this problem. This course does not intend to state that this procedure must be used, but if it is, this will provide the appraiser with an acceptable procedure for making the adjustment.
6. Steps for calculating absorption
    a. Calculate the NOI in the standard manner using market rent, market vacancy & collection, miscellaneous income (if applicable), and market expenses (expenses may be less due to reduced occupancy).
    b. Calculate the NOI again using the actual vacancy rate.
    c. Subtract the reduced NOI (2 above) from the standard NOI (1 above) for the rent loss.
    d. Add the following to the calculated rent loss:

Leasing costs: Commission rate x typical lease period x estimated market rent x Square footage that will be leased up in period to attain stabilized occupancy.

Tenant Improvement costs: Typical improvement cost per square foot necessary to attain market rent x square footage.

e. Multiply the Present Value factor at the appropriate discount rate x the sum of the rent loss + leasing costs + tenant improvement costs. This will be the present value of the cost to attain stabilized occupancy in one year.
f. If the appraiser believes that it will take two or more years for the property to lease up to market level occupancy, a second year (and so forth) calculation must be made using steps 1 through 5 above.
g. The actual vacancy used should be changed to a projected vacancy at the beginning of each year calculated for a declining amount over time. After all years are calculated, add all present values together for a final present value to be deducted from the final estimate of value by the income approach. Illustration 2-22

Absorption — short cut method

1. Calculate the square footage of the below market occupancy. This would be actual vacancy rate minus market vacancy rate (less collection rate) x total square footage of property.
2. Calculate the NOI rent rate. This would be the rent rate per square foot minus the operating expenses per square foot including property taxes if it is a gross or full service rate. For leases other than full service, you would need to add those costs that the owner would have to pay if the space were vacant which would include taxes and the other necessary operating expenses not deducted in the NOI calculation.
3. Determine the number of years it will take to lease up the property to the market occupancy level.
4. Determine the appropriate discount rate to calculate the present value

5. For the first year of lease up, multiply the NOI rent rate x the below market occupancy square footage x the year 1 present worth factor = present worth of the rent loss.
6. Repeat step e above for each additional year needed to complete the lease up period. Reduce the below market occupancy square footage in equal amounts for each year to achieve a declining vacant area (unless factual information would reveal otherwise). Also, each year would require a different present worth factor corresponding to the year number.
7. Add up all present worth values for a total present value of rent loss to be deducted from the final estimate of value by the income approach.
8. Example applications using spreadsheets are found on Illustration 2-23.

ILLUSTRATION 2-1

Future Value of $1 – Annual Compounding

What is the future value of $10,000 with an interest rate of 10% compounded annually for 10 years?

Using the table:
$10,000 x 2.593742 = $25,937.4200

Using the HP12C:

Step	Explanation	Key Stroke	Display
1	Solve for FV		
2	Clear memory	GOLD f CLX	0.0000
3	Enter number of periods	10 n	10.0000
4	Enter interest rate	10 i	10.0000
5	Enter PV	10,000 CHS PV	-10,000.0000
6	Enter PMT	0 PMT	0.0000
7	Calculate FV	FV	25,937.4246

ILLUSTRATION 2-2

Future Value of 1 – Annual Compounding

What is the future value of $10,000 with an interest rate of 10% compounded annually for 10 years and 7 months?

Using the table:
This will require interpolation of the table.
Factor for year 11 – factor for year 10 = 2.853117 – 2.593742 = 0.2594
This is for 12 months and there are only 7 so 7 ÷ 12 = 0.5833
0.2594 x 0.5833 = 0.1513 adjustment to year 10
2.593742 + 0.1513 = 2.7450 x $10,000 = $27,450.3544

Using the HP12C:

Step	Explanation	Key Stroke	Display
1	Solve for FV		
2	Clear memory	GOLD f CLX	0.0000
3	Enter number of complete years	10 n	10.0000
4	Save	STO 7	10.0000
5	Calculate 7 month factor	7 Enter 12 ÷	0.5833
6	Total factor	STO 7 + n	10.5833
7	Enter interest rate	10 i	10.0000
8	Enter PV	10,000 CHS PV	-10,000.0000
9	Enter PMT	0 PMT	0.0000
10	Calculate FV	FV	27,450.4410

Alternate method (Does not require STO)

Step	Explanation	Key Stroke	Display
1	Solve for FV		
2	Clear memory	GOLD f CLX	0.0000
3	7 months a percent	7 ÷ 12	.5833
4	Enter number of complete years	10 + n	10.5833
5	Enter interest rate	10 i	10.0000
6	Enter PV	10,000 CHS PV	-10,000.0000
7	Enter PMT	0 PMT	0.0000
8	Calculate FV	FV	27,450.4410

ILLUSTRATION 2-3

Future Value of 1 Per Period – Annual Compounding

How much money would be accumulated if there were annual deposits of $100 at an 8% interest rate for 10 years?

Using the tables:
$100 x 14.486562 = $1,448.6562

Using the HP12C;

Step	Explanation	Key Stroke	Display
1	Solve for FV		
2	Clear memory	GOLD f CLX	0.0000
3	Enter number of periods	10 n	10.0000
4	Enter interest rate	8 i	8.0000
5	Enter PMT	100 CHS PMT	-100.0000
6	Enter PV	0 PV	0.0000
7	Calculate FV	FV	1,448.6562

ILLUSTRATION 2-4

Future Value of 1 Per Period – Monthly Compounding

How much money would be accumulated if there were monthly deposits of $100 at an 8% interest rate for 10 years?

Using the tables:
$100 x 182.946035 = $18,294.6035

Using the HP12C:

Step	Explanation	Key Stroke	Display
1	Solve for FV		
2	Clear memory	GOLD f CLX	0.0000
3	Enter number of periods	10 g n	120.0000
4	Enter interest rate	8 g i	0.6667
5	Enter PMT	100 CHS PMT	-100.0000
6	Enter PV	0 PV	0.0000
7	Calculate FV	FV	18,294.6035

EXERCISE 2-1

Future Value – Annual Compounding

A property owner has decided to invest $12,000 annually into an account that earns 6% interest with annual compounding. What is the future value of the account 8 years from now if the contribution is made at the end of each year?

ILLUSTRATION 2-5

Sinking Fund – Annual Compounding

What annual deposit is required to accumulate $10,000 in 10 years at a 10% rate?

Using the tables:
$10,000 x 0.062745 = $627.4500

Using the HP12C:

Step	Explanation	Key Stroke	Display
1	Solve for PMT		
2	Clear memory	GOLD f CLX	0.0000
3	Enter number of periods	10 n	10.0000
4	Enter interest rate	10 i	10.0000
5	Enter FV	10,000 CHS FV	-10,000.0000
6	Enter PV	0 PV	0.0000
7	Calculate PMT	PMT	627.4539

ILLUSTRATION 2-6

Sinking Fund – Monthly Compounding

What monthly deposit is required to accumulate $10,000 in 10 years at a 10% rate?

Using the tables:
$10,000 x 0.004882 = $48.82

Step	Explanation	Key Stroke	Display
1	Solve for PMT		
2	Clear memory	GOLD f CLX	0.0000
3	Enter number of periods	10 g n	120.0000
4	Enter interest rate	10 g i	0.8333
5	Enter FV	10,000 CHS FV	-10,000.0000
6	Enter PV	0 PV	0.0000
7	Calculate PMT	PMT	48.8174

Using the same information, what would be the accumulated amount if the periods were made at the "Beginning of the Month"?

Step	Explanation	Key Stroke	Display
1	Solve for PMT		
2	Clear memory	GOLD f CLX	0.0000
3	Begin key. Begin (under the number 7 key) should show under the number display window	g 7	0.0000
4	Enter number of periods	10 g n	120.0000
5	Enter interest rate	10 g i	0.833
6	Enter FV	10,000 CHS FV	-10,000.0000
7	Enter PV	0 PV	0.0000
8	Calculate PMT	PMT	48.4140

Using the tables:
Year 9 factor – Year 10 factor: 0.005745 - 0.004882 = 0.000863
0.000863 ÷ 12 = 0.000072 for 1 month
Year 10 factor – 1 month: 0.004882 – 0.000072 = 0.004810
$10,000 x 0.004810 - $48.10
Answers will be slightly different because of compounding.

EXERCISE 2-2

Sinking Fund – Annual Compounding

An apartment complex needs to establish a sinking fund account for the roof that must be replaced in 8 years. If they make annual payments and can earn interest of 5.5% compounded annually, what amount must be contributed to reach a goal of $65,000?

ILLUSTRATION 2-7

Present Value of 1 – Annual Compounding

What is the present value of $30,000 to be received in 5 years at an interest rate of 8%?

Using the tables:
$30,000 x 0.680583 = $20,417.4959

Step	Explanation	Key Stroke	Display
1	Solve for PV		
2	Clear memory	GOLD f CLX	0.0000
3	Enter number of periods	5 n	5.0000
4	Enter interest rate	8 i	8.0000
5	Enter FV	30,000 CHS FV	-30.000.0000
6	Enter PMT	0 PMT	0.0000
7	Calculate PV	PV	20,417.4959

ILLUSTRATION 2-8

Present Value of 1 Per Period – Monthly Compounding

What is the present value of an ordinary level annuity of $1,000 per month for 10 years with an interest rate of 8%?

Using the tables:
$1,000 x 82.421481 = $82,421.48

Step	Explanation	Key Stroke	Display
1	Solve for PV		
2	Clear memory	GOLD f CLX	0.0000
3	Enter number of periods	10 g n	120.0000
4	Enter interest rate	8 g i	0.6667
5	Enter PMT	1,000 CHS PMT	-1,000.0000
6	Enter FV	0 FV	0.0000
7	Calculate PV	PV	82,421.4809

ILLUSTRATION 2-9

Present Value of 1 Per Period – Annual Compounding

The projected net income for each of the next four years is $23,800. If the desired rate of return is 9%, what is the present value of the level terminal annuity?

Using the tables:
$23,800 x 3.239720 = $77,105.3360

Using the HP12C:

Step	Explanation	Key Stroke	Display
1	Solve for PV		
2	Clear memory	GOLD f CLX	0.0000
3	Enter number of periods	4 n	4.0000
4	Enter interest rate	9 i	9.0000
5	Enter PMT	23,800 CHS PMT	-23,800.0000
6	Enter FV	0 FV	0.0000
7	Calculate PV	PV	77,105.3331

ILLUSTRATION 2-10

Present Value of 1 – Annual Compounding

What is the present value of $10,000 to be received in 10 years and seven months at an interest rate of 10% with annual compounding?

Step	Explanation	Key Stroke	Display
1	Solve for PV		
2	Clear memory	GOLD f CLX	0.0000
3	Enter number of complete years	10 n	10.0000
4	Save	STO 7	10.0000
5	Calculate 7 month factor	7 Enter 12 ÷	0.5833
6	Total factor	STO 7 + n	10.5833
7	Enter interest rate	10 i	10.0000
8	Enter FV	10,000 CHS FV	-10,000.0000
9	Enter PMT	0 PMT	0.0000
10	Calculate PV	PV	3,642.9287

Alternate keystrokes

Step	Explanation	Key Stroke	Display
1	Solve for FV		
2	Clear memory	GOLD f CLX	0.0000
3	7 months at percent	7 ENTER 12 ÷	.5833
4	Enter number of complete years	10 + n	10.5833
5	Enter interest rate	10 i	10.0000
6	Enter PV	10,000 CHS FV	-10,000.0000
7	Enter PMT	0 PMT	0.0000
8	Calculate FV	PV	3,642.9287

EXERCISE 2-3

Present Value – Annual Compounding

What is the present value of the right to receive $23,000 per year for 5 years at an interest rate of 7%?

ILLUSTRATION 2-11

Partial Payment – Full Amortization

What is the level monthly payment for a direct reduction loan in the amount of $150,000 with an interest rate of 8% and full amortization in 25 years?

Using the tables:
$150,000 x 0.007718 = $1,157.7000

Using the HP12C:

Step	Explanation	Key Stroke	Display
1	Solve for PMT		
2	Clear memory	GOLD f CLX	0.0000
3	Enter number of periods	25 g n	300.0000
4	Enter interest rate	8 g i	0.6667
5	Enter PV	150,000 CHS PV	-150,000.0000
6	Enter FV	0 FV	0.0000
7	Calculate PMT	PMT	1,157.7243

ILLUSTRATION 2-12

Partial Payment & Loan Balance

What is the level monthly payment for a direct reduction loan in the amount of $150,000 with an interest rate of 8% and full amortization in 25 years and what would be the remaining loan balance after 10 years?

Step	Explanation	Key Stroke	Display
1	Solve for PMT		
2	Clear memory	GOLD f CLX	0.0000
3	Enter number of periods	25 g n	300.0000
4	Enter interest rate	8 g i	0.6667
5	Enter PV	150,000 CHS PV	-150,000.0000
6	Enter FV	0 FV	0.0000
7	Calculate PMT	PMT	1,157.7243
8	Enter years	10 g n	120.0000
9	Calculate loan balance	FV	121,144.9594

Proof:
$1,157.7243 payable monthly for 15 years has a present value $121,144.9594

Step	Explanation	Key Stroke	Display
1	Solve for PV		
2	Clear memory	GOLD f CLX	0.0000
3	Enter number of periods	15 g n	180.00
4	Enter interest rate	8 g i	.6666
5	Enter PMT	1,157.7243 CHS PMT	-1,157.7243
6	Enter FV	0 FV	0.0000
7	Calculate PV	PV	121,144.9594

Using the tables:
$150,000 x 0.007718 (Partial payment) = $1,157.70
$1,157.70 x 104.640592 (Present worth of 1/Period for 15 years) = $121,142.41

ILLUSTRATION 2-13

Mortgage Constant

What would be the annual mortgage constant term for a 30-year loan at a 12% interest rate with monthly payments?

Step	Explanation	Key Stroke	Display
1	Solve for mortgage constant		
2	Clear memory	GOLD f CLX	0.0000
3	Enter number of periods	30 g n	360.0000
4	Enter interest rate	12 g i	1.0000
5	Enter PV	1 CHS PV	-1.0000
6	Enter FV	0 FV	0.0000
7	Calculate monthly constant	PMT	0.0103
8	Calculate annual mortgage constant	12 x	0.1234

The monthly partial payment factor in Appendix 2M is 0.010286. If the payment is monthly, the partial payment factor from the monthly table can be multiplied by 12 to find the annual mortgage constant: 0.010286 x 12 = 0.123432.

EXERCISE 2-4

Mortgage Constant

What would be the annual mortgage constant term for a 15-year loan at an 8.0% interest rate with monthly payments?

ILLUSTRATION 2-14

Leased Fee Estate

There are 10-years remaining on a commercial lease. The contract rent is $12,000 per year. You believe the property will be worth $300,000 at the end of the lease.

What is the value of the leased fee estate, using a yield rate of 12%?

Using the tables:
Present worth of 1/period factor = 5.650223
Present worth of 1 factor = 0.321973
$12,000 x 5.650233 = $67,802.7960
$300,000 x 0.321973 = $96,591.9000
Total = $164,394.6960

Using the HP12C:

Step	Explanation	Key Stroke	Display
1	**Solve for Present Value of Cash Flows**		
2	Clear memory	GOLD f CLX	0.0000
3	Enter number of periods	10 n	10.0000
4	Enter interest rate	12 i	12.0000
5	Enter payment	12,000 CHS PMT	-12,000.0000
6	Enter future value	0 FV	0.0000
7	Solve for present value	PV	67,802.6763
8	Save	STO 7	67,802.6763
9	**Solve for Reversion**		
10	Enter future value	300,000 CHS FV	-300,000.0000
11	Change PMT to zero	0 PMT	0.0000
12	Solve for present value of reversion	PV	96,591.9710
13	Recall stored cash flow amount	RCL 7	67,802.6763
14	Add reversion and cash flows	+	164,394.6473

ILLUSTRATION 2-15

Sandwich Lease Value

Assume the site is subleased at $15,000 per year, what is the value of the sandwich leasehold at 15%?

Using the tables:
Present worth of 1/Period factor = 5.018769
$15,000 - $12,000 = $3,000
$3,000 x 5.018769 = $15,056.3070

Using the HP12C;

Step	Explanation	Key Stroke	Display
1	**Solve for Leased Fee Estate**		
2	Clear memory	GOLD f CLX	0.0000
3	Enter subleased amount	15,000 ENTER	15,000.0000
4	Enter contract rent	12,000 -	3,000.0000
5	Enter sandwich leasehold	CHS PMT	-3,000.0000
6	Enter interest rate	15 i	15.0000
7	Enter number of periods	10 n	10.0000
8	Solve for present value	PV	15,056.3059

What is the value of the fee simple estate?

The market rent must be known. If the actual rent is equal to market rent, then the leased fee estate would equal the fee simple estate.

ILLUSTRATION 2-16

DCF and Property Taxes

Real Estate Taxes - are a valid operating expense of property. However, because the ad valorem tax appraiser is appraising the property for the ultimate purpose of determining the amount of ad valorem taxes, the appraiser must not make a deduction for real estate taxes. It is not possible to know the proper amount of real estate taxes until the amount of the assessed value is known. Thus, to make a deduction for real estate taxes, the appraiser would have to pre-suppose the amount of value in order to compute the amount of real estate taxes. The ad valorem tax appraiser circumvents this problem by making no deduction for real estate taxes from the income and instead the appraiser includes the effective tax rate as a component of the capitalization rate for the property.

Use of discounted cash flow recognizes the investor's perspective and investors would consider property taxes as an expense. How to recognize the property taxes in discounted cash flow creates a dilemma for the ad valorem appraiser. This dilemma is resolved by loading the discount (property yield) rate with the effective tax rate. This is similar to loading the capitalization rate covered earlier. In other words, if the property yield rate YO is say 8% and the effective tax rate is 2% the cash flows before deduction for property taxes could be discounted at 10%. This technique allows the appraiser to recognize property taxes as an expense without pre-concluding the value of the property.

A Discounted Cash Flow Model with Income, Expense and Value Changing at Constant Rates

Appraisal Conditions:		**Enter Data**
Forecast Period (Yrs.)		10
Desired Annual Yield Rate		9.00%
Nominal Tax Rate (in mills)		20
Assessment Level (% of MV)		100%
Annual change in PGI		3%
Year 1 V&C % of PGI		5%
Annual % Change in V&C		3%
Year 1 Expense as % of EGI		20%
Annual Change in Exp.		3%
Annual Change in ETR		0%
Terminal Cap Rate (R_n)		7.60%

ILLUSTRATION 2-16 – DCF and Property Taxes Continued

Notes:
(1) Expenses – Not including property taxes.
(2) NOI – Net operating income before deducting property taxes.
(3) Reversion – Reversion based on NOI in 11th year capitalized at market R_o plus effective tax rate at time of resale, less estimated resale costs @ 5%. The calculated reversion is shown column 11. How the calculations work on shown on the next page.
(4) Discount Factor – Discount factor adjusted for property tax load (investment yield rate plus effective tax rate. The present value factor would then be for 11% (yield of 9% plus effective tax of 2%)

Year	1	2	3	4	5	6	7	8	9	10	11
PGI	$84,000	$86,520	$89,116	$91,789	$94,543	$97,379	$100,300	$103,309	$106,409	$109,601	$112,889
V&C Loss	$4,200	$4,326	$4,456	$4,589	$4,727	$4,869	$5,015	$5,165	$5,320	$5,480	-5,644
EGI	$79,800	$82,194	$84,660	$87,200	$89,816	$92,510	$95,285	$98,144	$101,088	$104,121	$107,245
Exp (1)	$15,960	$16,439	$16,932	$17,440	$17,963	$18,502	$19,057	$19,629	$20,218	$20,824	-21,449
NOI (2)	$63,840	$65,755	$67,728	$69,760	$71,852	$74,008	$76,228	$78,515	$80,871	$83,297	$85,796
Cash Flow	$63,840	$65,755	$67,728	$69,760	$71,852	$74,008	$76,228	$78,515	$80,871	$83,297	÷ 0.096
Dis. Factor (3)	0.9009	0.81162	0.73119	0.65873	0.59345	0.53464	0.48166	0.43393	0.39092	0.35218	$893,708
DCF	$57,514	$53,368	$49,522	$45,953	$42,641	$39,568	$36,716	$34,070	$31,614	$29,336	-44,685
Reversion											$849,023
NPV	$719,310										0.35218
Rounded =	**$719,000**										$299,008

ILLUSTRATION 2-16 – DCF and Property Taxes (Continued)

Calculation of the reversion is shown below.

Year 10 Potential Gross income x 1.03	$ 112,889
Year 10 Vacancy & collection loss x 1.03	- 5,644
Year 11 Effective gross income	$107,245
Year 10 Expenses x 1.03	- 21,449
Year 11 Net operating income	$ 85,796
Year 11 Capitalization rate; Going-in 0.076 + Effective tax rate 0.02	÷ 0.096
Year 11 Value indication	$893,708
Year 11 Sales commission: Value x 0.05	- 44,685
Reversionary value	$849,023
Present value factor of reversion	0.35218
NPV of the Reversionary value	$299,008

Calculation of the going in rate is shown below

Quick Check	1st yr NOI ÷	NPV	=	R_o + ETR
	$63,840	$719,000	=	8.879%
ETR =	2.00%		-	2.00%
Tax =	-$14,380		R_o	6.8790%
NOI after tax	$49,460	$719,000	=	6.8790%

Forecast Results:

Effective Tax Rate (ETR)		2.00%
Adjusted Discount Rate		11.00%
Adjusted Terminal Cap Rate (1)		9.60%
Net Present Value		$719,000
Year 1 Property Taxes (2)		$14,380
Going in Cap Rate R_O (3)		6.88%

Notes:
(1) Adjusted Terminal Cap Rate – Terminal rate plus effective tax rate. (0.076 + 0.02).
(2) Year 1 Property Taxes – NPV times ETR.
(3) Going in Cap Rate R_o – Calculated in the previous box as NOI after tax.

ILLUSTRATION 2-16 – DCF and Property Taxes Continued

Thoughts on how to handle property taxes in DCF vary greatly. A check follows that supports the application just shown.

Notes:
(1) Expenses – Expenses not including property tax.
(2) NOI – NOI after deducting property tax.
(3) Reversion – Reversion based on NOI for 11th year capitalized at market R_o (not loaded for taxes) at time of resale, less estimated resale costs @ 5%.
(4) Discount Factor – Discount factor based on market yield (this factor does not include a load for taxes).
(5) Difference – This is the percentage difference between the DCF model loaded for the effective tax rate and the model used as a check with taxes as a expense. [($721,000 - $719,000) ÷ $719,000] = 0.0028 x 100 = 0.28%.

Checking Taxes Changing @ 2%

	1	2	3	4	5	6	7	8	9	10
PGI	$84,000	$86,520	$89,116	$91,789	$94,543	$97,379	$100,300	$103,309	$106,409	$109,601
V&C Loss	$4,200	$4,326	$4,456	$4,589	$4,727	$4,869	$5,015	$5,165	$5,320	$5,480
EGI	$79,800	$82,194	$84,660	$87,200	$89,816	$92,510	$95,285	$98,144	$101,088	$104,121
Exp (1)	$15,960	$16,439	$16,932	$17,440	$17,963	$18,502	$19,057	$19,629	$20,218	$20,824
Prop. Tax	**$14,380**	**$14,668**	**$14,961**	**$15,260**	**$15,565**	**$15,877**	**$16,194**	**$16,518**	**$16,848**	**$17,185**
NOI (2)	$49,460	$51,088	$52,767	$54,500	$56,287	$58,131	$60,034	$61,997	$64,022	$66,111
Reversion (3)	$0	$0	$0	$0	$0	$0	$0	$0	$0	$853,331
Cash Flow	$49,460	$51,088	$52,767	$54,500	$56,287	$58,131	$60,034	$61,997	$64,022	$919,442
Dis. Factor (4)	0.91743	0.84168	0.77218	0.70843	0.64993	0.59627	0.54703	0.50187	0.46043	0.42241
DCF	$45,376	$43,000	$40,746	$38,609	$36,583	$34,662	$32,840	$31,114	$29,478	$388,382
NPV	$720,790									
Rounded =	$721,000									
Difference (5)	0.28%									

NOI (2) Annual Change =	**3.30%**	**3.30%**	**3.30%**	**3.30%**	**3.30%**	**3.30%**	**3.30%**	**3.30%**	**3.30%**	

EXERCISE 2-5

Calculation of R_o With Property Tax

Calculate the first year's NOI and what the total capitalization rate would be for year 3 using four places to the right of the decimal. Then, if the NOI for Year 3 is the same as for Year 1, calculate the market value.

PGI = $318,000 Vacancy & collection loss = 6%
Expenses not including property tax = 20%
Effective tax rate = 3.7% Yield rate = 8.5%
Taxes are increasing by 2% per year

ILLUSTRATION 2-17

Rent Projections

An example of a 1,200 square foot Post Office below shows not only rent increases but the bumps or escalations in the rent based upon the lease agreement.

Year	Annual Gross Rent	Rent / SF
1	$8,300	$6.91
2	$9,270	$7.73 (1)
3	$9,270	$7.73
4	$9,270	$7.73
5	$9,270	$7.73
6	$10,440	$8.70 (2)

Using the Post Office rent from above, the variable annuity (rent) could be calculated as follows.

Year 1 Rent	$6.91 per SF
Years 2 - 5 Rent	$7.73 per SF 11.9% Increase
Years 6 – 9 Rent	$8.70 per SF 12.6% Increase
Estimate Year 10 Rent	$9.74 per SF 12.0% Increase

(1) (2) – This would indicate that the lease is graduated with step-ups each four years.

Most leases would state a monthly rent and again an annual increase. An example of three years is shown below with a 3% annual compounded increase.

Year 1 Rent	$4,000.00 per Month
Year 2 Rent	$4,120.00 per Month ($4,000 x 1.03)
Year 3 Rent	$4,243.60 per Month ($4,120 x 1.03)

ILLUSTRATION 2-18

Internal Rate of Return

INITIAL INVESTMENT = $ 90,000.

DESIRED RATE OF RETURN = 10%

REVERSION = $ 100,000

USING THE CASH FLOW INFORMATION BELOW, WHAT WOULD BE THE INTERNAL RATE OF RETURN?

TIME	CASH FLOW
0	- $ 90,000
1	$ 5,000
2	$ 25,000
3	$ 15,000
4	$ 3,000
5	$ 10,000
5	$100,000

***Continued on Next Page

ILLUSTRATION 2-18 – Internal Rate of Return (Continued)

Step	Explanation	Key Stroke	Display
1	Solve for IRR		
2	Clear memory	GOLD f CLX	0.0000
3	Enter initial investment	90,000 CHS g CFo	-90,000.0000
4	Enter annual cash flow	5,000 g CFj	5,000.0000
5	Enter annual cash flow	25,000 g CFj	25,000.0000
6	Enter annual cash flow	15,000 g CFj	15,000.0000
7	Enter annual cash flow	3,000 g CFj	3,000.00
8	Enter annual cash flow	110,000 g CFj	110,000.0000
9	Number of periods	RCL n	5.0000
10	Desired yield	10 i	10.0000
11	Solve for net PV	f NPV	16,826.7195
12	Solve for IRR	f IRR	14.8434

General Comments:

1. The number of periods step is not required but helps to verify the number of years of cash flow and assures that the memory was cleared.

2. The gold HP12C will take considerable time to calculate the IRR but the newer silver HP12C is very fast.

3. In the last year the income and reversion must be combined into one amount or the calculator will treat one as occurring in the next year.

4. If it is an annuity in advance, then switch the calculator from END to BEGIN. If using the manual tables, then PV for Year 1 is 1.00 and then for Year 2 it is the PV factor for Year 1 and so on. Calculating IRR without an HP12C or other financial calculator is a try and error routine.

5. Process can be worked with a typical calculator by use of Appendix [illegible].

ILLUSTRATION 2-19

Internal Rate of Return

INITIAL INVESTMENT = $ 90,000.

DESIRED RATE OF RETURN = 10%

REVERSION = $ 100,000

USING THE CASH FLOW INFORMATION BELOW, WHAT WOULD BE THE INTERNAL RATE OF RETURN?

Year	Amount
Initial outlay of cash	- $ 90,000
1	$ 5,000
2 & 3 each	$ 25,000
4 & 5 each	$ 15,000
6	$ 3,000
7	$ 10,000
7 – Reversion	$100,000

Step	Explanation	Key Stroke	Display
1	Solve for IRR		
2	Clear memory	GOLD f CLX	0.0000
3	Enter initial investment	90,000 CHS g CFo	-90,000.0000
4	Enter annual cash flow	5,000 g CFj	5,000.0000
5	Enter annual cash flow	25,000 g CFj	25,000.0000
6	Number of same cash flows	2 g Nj	2.0000
7	Enter annual cash flow	15,000 g CFj	15,000.0000
8	Number of same cash flows	2 g Nj	2.0000
9	Enter annual cash flow	3,000 g CFj	3,000.00
10	Enter annual cash flow	110,000 g CFj	110,000.0000
11	Number of periods	RCL n	5.0000
12	Desired yield	10 i	10.0000
13	Solve for net PV	f NPV	31,689.3181
14	Solve for IRR	f IRR	17.2294

ILLUSTRATION 2-20

Calculating Present Value

The annual net income is $8,000 per year for three years and at the end of the three years the reversion or selling price will be $65,000 and the yield rate is 9%.

Four methods of calculation will be shown. The first two are manual by looking up the factors from the present worth tables and the last two will show the steps using the HP12C. It is common that most DCF will be conducted by use of a computerized spreadsheet, but how the calculations work must be understood. PW 1 is the Present Worth of One table and the PW 1/P is the Present Worth of One Per Period table. The Present Worth of 1, "also called the reversion factor. The lump-sum amount that would have to be set aside to accumulate with compound interest to $1 at the end of a specified number of years and at a specified rate of interest," IAAO Glossary. The Present Worth of 1 per Period, "also called the annuity factor or Inwood coefficient. The present worth of a series of payments of $1, receivable at the end of each year, for a specified number of years and at a specified interest rate," IAAO Glossary.

Year	PW 1	PW 1/Period
1	0.917431	0.917431
2	0.841680	1.759111
3	0.772183	2.531295
4	0.708425	3.239720
5	0.649931	3.889651

Method #1: (See Appendix 2)

$ 8,000 x 0.917431 = $ 7,339		$ 8,000 x 0.917431 = $ 7,339
$ 8,000 x 0.841680 = $ 6,733	OR	$ 8,000 x 0.841680 = $ 6,733
$ 8,000 x 0.772183 = $ 6,177		$73,000 x 0.772183 = $ 56,370
$65,000 x 0.772183 = $ 50,192		Present Worth $70,442
Present Worth $ 70,441		

This method uses the factor for each year times the net income to calculate an annualized present worth and the same for the reversion.

ILLUSTRATION 2-20 – Calculating Present Value (Continued)

Method #2:
$ 8,000 x 2.531295 = $ 20,250
$65,000 x 0.772183 = $ 50,191
Present Worth $ 70,441

This method recognizes a level annuity for the three years and thus can use the Present Worth of 1/Period for the rental income and then calculates the reversion with the factor when it is received at the end of the third year. If the annuity is not level, then only Method #1 can be used.

There are also several ways to calculate the answer by use of the HP12C.

Step	Explanation	Key Stroke	Display
1	Clear memory	GOLD f CLX	0.0000
2	Number of periods	3 n	3.0000
3	Enter interest rate	9 i	9.0000
4	Enter income	8000 CHS PMT	-8,000.0000
5	Enter FV	0 FV	0.0000
6	Calculate PV	PV	20,250.3573
7	Reversion	3 n	3.0000
8	Interest rate	9 i	9.0000
9	Enter reversion	65,000 CHS FV	65,000.0000
10	Enter PMT	0 PMT	0.0000
11	Calculate PV	PV	50,191.9262
12	Total PV	+	70,442.2835

The application below would be used when the income is not a level annuity.

Step	Explanation	Key Stroke	Display
1	Clear memory	GOLD f CLX	0.0000
2	Enter income year 1	8,000 g CFj	8,000.0000
3	Enter income year 2	8,000 g CFj	8,000.0000
4	Enter income and reversion for year 3	73,000 g CFj	73,000.0000
5	Enter interest rate	9 i	9.0000
6	Calculate PV	f NPV	70,442.2835

EXERCISE 2-6

Net Present Value

A. NOI: Year 1 = $14,600, Year 2 = $15,100, Year 3 = $16,000 Year 4 = $17,200 plus a reversion of $400,000. What is the NPV if the yield rate is 9%?

B. What is the NPV using the same information except that the yield rate for the reversion is 11%?

Review Questions

REVIEW QUESTIONS

1. Discounted cash flow uses which capitalization rate _____.

2. The length of time the investor keeps a property is called the
_____ _____

3. Converting future income into present value is the process of
_____

4. Selling the property at the end of the holding period is a term called
_____

5. Name two types of properties that discounted cash flow could be used for determining market value _____ _____

6. Making projections of the income and expenses is called
_____

7. What compound interest factor is used to convert the reversion into present value _____

8. What is the right to receive money or its equivalent at regular intervals for a definite or indefinite term _____

Given the following information, answer Questions 9 & 10.
NOI: Year 1 = $12,200, Year 2 = $13,400, Year 3 = $14,600
Reversion: $125,000 Yield Rate: 10%

9. What is the NPV of year 1 _____

10. What is the indicated net present value _____

11. The rate used to calculate the reversion is referred to as the
_____ rate

12. In DCF analysis, what four factors must be determined for the cash flows
_____ _____ _____ _____

13. It is generally recognized that the terminal rate used to calculate the reversion may be _____ than the rate used for the income stream.

14. A yield capitalization method used to calculate the present value of anticipated future cash flows defines _____

_____ _____

15. Name a weakness of DCF _____

16. The first step in calculating below normal vacancy is

_____

17. The rate that measures the real return on investment after the reversion is the _____ _____ ___ _____

18. What annual deposit is required to accumulate $30,000 in 10 years at a 6% rate?

The following information is for Questions 19 & 20.
The projected net income for the each of the next five years is $61,000, with a reversion of $700,000 at the end of year five and the desired rate of return is 9%.

19. What is the net present value of the level terminal annuity? _____

20. What is the total present value? _____

Using the following information, answer questions 21 & 22.
NOI before property taxes is $360,000 for year 1, $390,000 for year 2 and $410,000 for year 3. The effective tax rate is 1.8% and is anticipated to remain the same for each year. At the end of year 4, it is anticipated the property will sell for $4,600,000. The yield rate is 9.5%. PV factors for the income stream are:

0.898473 0.807253 0.725295

Reversion PV factor is 0.651657

21. What is the NPV for the annuity of year 3? _____

22. What is the NPV? _____

23. What is the ratio between EGI and the property expenses including debt service? _____ _____

Quiz

Quiz

1. The type of lease clause that shifts increases in future expenses from the landlord to the tenant

   a. Escalation
   b. Reappraisal
   c. Net
   d. Percentage

2. The area within a building or structure that is actually occupied by an individual tenant

   a. Net leaseable area
   b. Gross leaseable area
   c. Net contract area
   d. Gross useable area

The following information is for questions 3 &4. A property is being leased for $7.80 per square foot of net leaseable area of 3,500 square feet plus 3% of sales greater than $3,750,000 per year. Annual sales last year were $4,648,500.

3. What is the base rent?

   a. $ 34,500
   b. $ 27,300
   c. $ 26,955
   d. $ 54,255

4. What is the overage rent?

   a. $ 34,500
   b. $ 27,300
   c. $ 26,955
   d. $ 54,255

5. Land area and shape fall under which of the four tests for highest and best

   a. Physically possible
   b. Legally permitted
   c. Financially feasible
   d. Maximally productive

6. A loss in income for the owner because of an agreement with the tenant to either reduce the rent on short term basis, or allow free rent for a short term

   a. Rent concessions
   b. Tenant improvement
   c. Excess vacancy
   d. Broker's fee

The following are for questions 7 & 8. Base year expenses are $435,000 and current year expenses are $571,346. You are leasing 5,000 SF of the total NLA of 80,000.

7. What is the amount of your pass-through expenses?

   a. $12,235
   b. $ 8,522
   c. $14,098
   d. $37,654

8. What is the expense stop per SF?

   a. $5.44
   b. $7.14
   c. $6.90
   d. $3.67

9. The investment time is also referred to as

   a. Equity
   b. Holding period
   c. Loan period
   d. Leveraging

10. In order to fill the vacant space, a property owner will have to make TI of $126,500 to the NLA of 5,200 SF. The previous tenant was paying $18.00 per SF and was considered market. If the new lease will be for 6 years, what will be the necessary rent to recover the cost of the TI?

    a. $18.00
    b. $22.05
    c. $25.80
    d. $20.00

11. Changes as called for in the lease agreement at one or more points during the lease term is also known as

    a. Long-term
    b. Percentage
    c. Graduated
    d. Absolute net

12. The rate used at the end of the holding period to convert reversionary value is the
    a. Discount rate
    b. Terminal rate
    c. Recovery rate
    d. Mortgage equity rate

13. What factor is also referred to as the reversion?

    a. Present worth of 1
    b. Present worth of 1 per period
    c. Sinking fund
    d. Inwood

Given the following information, calculate the PV of the reversion and the total net property value. NOI Year 1 = $31,000, Year 2 = $34,000, Year 3 = $38,500 and reversion = $500,000. Yield rate is 9%.

14. Reversionary value is

    a. $372,000
    b. $468,000
    c. $416,000
    d. $386,000

15. Total NPV is

    a. $372,000
    b. $473,000
    c. $416,000
    d. $386,000

16. What would be the mortgage payment for a 20 year loan at a 7.5% interest rate with annual payments if the loan-to-value ratio is 80% with a purchase price of $1,600,000?

    a. $123,567
    b. $125,558
    c. $234,590
    d. $165,340

17. In DCF analysis the terminal rate is often _____ in relation to the yield rate because of the future value estimation.

    a. One-percent difference
    b. Lower
    c. Higher
    d. Not used with DCF

18. Effective gross income / 1- (Expenses + Annual debt service) is the formula for

    a. Breakeven point
    b. Yield change
    c. Mortgage constant
    d. Debt coverage ratio

19. The investor's internal rate of return is the

    a. Yield rate
    b. Recapture rate
    c. Overall rate
    d. Mortgage rate

20. If the investment is not totally recaptured between the income stream and the reversion, then the yield rate is actually

    a. Normal
    b. Positive
    c. Negative
    d. As projected

21. A loan was provided for $500,000 with an annual mortgage constant of 0.104628. The total sale price was $625,000 and has an operating expense ratio of 35% with NOI of $56,250, what would be the Ro?

    a. 10.4%
    b. 9.0%
    c. 7.3%
    d. 6.5%

22. The projected net income for the each of the next five years is $34,200. If the desired rate of return is 9%, what is the net present value of the level terminal annuity with a reversion of $400,000?

    a. $318,340
    b. $587,667
    c. $392,998
    d. $534,347

23. How long the income stream will last is

    a. Quantity
    b. Variability
    c. Timing
    d. Duration

24. Investors have indicated they expect an 11% overall yield (Y_o) for their investment and that the anticipated income and value increases over the next few years is 1.5%. What is the overall rate (R_o)?

a. 9.5%
b. 12.5%
c. 11.0%
d. 8.0%

25. An apartment complex needs to establish a sinking fund account for the roof that must be replaced in 6 years. If they make annual payments and can earn interest of 7.0% compounded annually, what amount must be contributed to reach a goal of $75,000?

a. $12,000
b. $8,000
c. $10,500
d. $9,300

Appendix I - Glossary

Glossary and Rates and Relationships

IRV

$$I = R \times V$$
$$R = I \div V$$
$$V = I \div R$$

VIF

$$V = I \times F$$
$$I = V \div F$$
$$F = V \div I$$

EAT

$$E = A \times T$$
$$A = E \div T$$
$$T = E \div A$$

$$R_O = Y_O - CR$$
$$Y_O = R_O + CR$$
$$CR = Y_O - R_O$$

$$DCR = NOI \div I_M$$

$$R_O = DCR \times M \times R_M$$

$$R_O = NIR \div EGIM$$

$$R_O = M \times R_M + (1 - M) \times R_E$$

$$R_L = I_L \div V_L$$

$$R_B = I_B \div V_B$$

$$R_E = I_E \div V_E$$

$$R_M = I_M \div V_M$$

$$NIR = 1 - OER$$

$$OER = 1 - NIR$$

GIM = Property Value ÷ Gross Income

EGIM = Property Value ÷ Effective Gross Income

Description of Various Rates

Overall Capitalization Rate (R_O) – The R_O reflects the relationship between a single years's net operating income and the total property value. It is typically derived from market comparisons with highly comparable sales, band of investment calculations, the DCR technique, NIR ÷ GIM, yield change techniques. It also reflects the weighted average of the R_L and R_B weighted by the respective contributions to value of the land and building. R_O can be either greater than, equal to, or less than Y_O depending on whether the property value is expected to decrease, remain stable, or increase respectively.

Overall Yield Rate (Y_O) – Sometimes referred to as the property discount rate or "r" or the "return on" rate, internal rate of return, or property interest rate. The overall yield rate is expressed as a compound annual percentage rate. It considers all expected benefits of property ownership, including the proceeds from the sale at the termination of the investment. It can be obtained by extraction from sales of similar properties, comparison with alternative investments of comparable risk, surveys of property investors, or the yield change formula – $Y_O = R_O + CR$. [CAUTION: Using the band of investment technique to blend Y_M and Y_E is mathematically incorrect unless the loan is "interest only" and there is no change in income or value over the holding period. Conceptually, the Y_O is a weighted average of the equity yield rate and the mortgage interest rate. However, in practice the Y_O is not usually calculated by the BOI technique because the ratio of debt and equity changes each year as the loan is amortized and the property value changes.]

Land Capitalization Rate (R_L) – The R_L reflects the relationship between a single year's net income attributable to the land and the value of the land. It is usually extracted from sales using the relationship $R_L = I_L \div V_L$. R_L can be either greater than, equal to, or less than Y_L depending on whether the land value is expected to decrease, remain stable, or increase respectively. [Note: Within the context of classic straight line capitalization where land income and value are not expected to change over the holding period, the land capitalization rate (including a property tax component) is the sum of the Y_O and the effective tax rate (because in that situation $R_L = Y_L$).]

Land Yield Rate (Y_L) – The Y_L reflects the rate of return on capital invested in the land portion of real property. It is expressed as a compound annual rate and considers all expected benefits of land ownership, including the proceeds from the sale of the land at the termination of the investment. Since $Y_L = Y_O = Y_B$ is virtually always true (because investors rarely assign a different risk rate to the land and building), the same methods used to derive Y_O are valid for deriving Y_L.

Building (Improvement) Capitalization Rate (R_B) – The R_B reflects the relationship between a single year's net income attributable to the building and the value of the building. It is usually extracted from sales using the relationship $R_B = I_B \div V_B$. R_B can be either greater than, equal to, or less than Y_B depending on whether the building value is expected to decrease, remain stable, or increase respectively. [Note: Within the context of classic straight line capitalization where building "return on" income is earned only on the undepreciated balance of the building investment during the holding period, the building capitalization rate (including a property tax component) is the sum of the Y_O, the recapture rate, and the effective tax rate (because in that situation, $R_B = Y_B$).]

Building (Improvement) Yield Rate (Y_B) – The Y_B reflects the rate of return on capital invested in the building portion of real property. It is expressed as a compound annual rate and considers all expected benefits of building ownership, including the proceeds from the sale of the building at the termination of the investment. Since $Y_B = Y_O = Y_L$ is virtually always true (because investors rarely assign a different risk rate to the land and building), the same methods used to derive Y_O are valid for deriving Y_B.

Mortgage Capitalization Rate (R_M) – Also known as the mortgage annual constant, the R_M is the ratio of the annual debt service (principal & interest). It reflects the relationship between mortgage income (I_M) and mortgage principal. It is one of the components used in the band of investment technique to get an R_O. It can be obtained from the compound interest tables by multiplying the partial payment factor (Column #6) by the number of conversion periods in one year.

Mortgage Yield Rate (Y_M) – The Y_M is also known as the mortgage interest rate. It is the lender's rate of return on a loan (if no points or other charges increase the lender's yield). It is usually obtained by interviewing lenders or can be extracted from the terms of the mortgage.

Equity Capitalization Rate (R_E) – The R_E is also known as the equity dividend rate or the cash on cash rate. It reflects the relationship between a single year's equity income and the value of the equity. It is typically extracted from sales using the relationship $R_E = I_E \div V_E$. It is extremely sensitive to loan-to-value ratios. [CAUTION: Unlike the Y_E, the R_E should not be derived from sales of stocks or bonds because of dissimilarity of investments, i.e. liquidity, holding term, etc.]

Equity Yield Rate (Y_E) – The Y_E is the "return on" rate for equity and is sometimes referred to as the equity discount rate or the equity interest rate. It considers all the expected cash flows attributable to the equity investment, including proceeds from sale at the termination of the investment. The Y_E can be extracted from sales of similar properties, from alternative investments of comparable risk such as stocks and bonds, or from surveys of market participants.

Loan to Value Ratio (M) – This is the ratio between a mortgage loan and the value of the property, usually expressed as a percentage. It is usually obtained from interviews with active lenders. It can be extracted from sales by dividing the loan amount by the selling price.

Debt Coverage Ratio (DCR) – This is the ratio of net operating income to annual debt service (principal & interest). It measures the ability of a property to meet its annual debt requirements out of net operating income. The DCR can be extracted from sales by dividing the net operating income by the annual debt service. The DCR is an ideal test of reasonableness and can be multiplied by the R_M and the M to obtain R_O. ($R_O = DCR \times R_M \times M$)

Effective Tax Rate (ETR) – This is the ratio of property taxes to total property value. It can be derived by the EAT formula by multiplying the assessment level times the tax rate as long as all components are expressed as proper decimals. ($ETR = A \times T$) It can also be extracted from sales by dividing the income to property taxes by total property value. It can be directly added to or subtracted from the R_O to get an R_O with or without a property tax component. Conceptually, the ETR cannot be directly added to or subtracted from the Y_O to get a Y_O with or without a property tax component because Y_O is a yield rate used in discounting the cash flows in yield capitalization and the ETR is a capitalization rate. An exception to this is when using classic straight line capitalization where the $Y_O = R_L$.

Operating Expense Ratio (OER) – The OER reflects the relationship between operating expenses and the Effective Gross Income (EGI) or between the relationship between operating expenses and Potential Gross Income. Consistency is paramount to using this ratio properly. If the OER is represented to be a percentage of EGI it must be applied consistently with the manner in which it was derived. Likewise, if the OER is represented to be a percentage of Potential Gross Income (PGI) it must be applied consistently with the manner in which it was derived.

Net Income Ratio (NIR) – The NIR reflects the relationship between net operating income and the Effective Gross Income (EGI) or between the relationship between net operating income and Potential Gross Income (PGI). Consistency is paramount to using this ratio properly. If the NIR is represented to be a percentage of EGI it must be applied consistently with the manner in which it was derived. Likewise, if the NIR is represented to be a percentage of (PGI) it must be applied consistently with the manner in which it was derived.

Abbreviations Used in Income Approach to Value

BOI	Band of Investment
CR	Constant Rate of Change
DCR	Debt Coverage Ratio
EAT	Effective Tax Rate, Assessment Level, & Tax Rate formula
EGI	Effective Gross Income
EGIM	Effective Gross Income Multiplier
ETR	Effective Tax Rate
GIM	Gross Income Multiplier (sometimes known as Gross Rent Multiplier)
GLA	Gross Leasable Area
GRM	Gross Rent Multiplier (sometimes known as Gross Income Multiplier)
I_B	Income to the Building (Improvement)
I_E	Income to the Equity
I_L	Income to the Land
I_M	Income to the Mortgage (income necessary to pay principal & interest)
IRV	Income Rate & Value formula
I_V	Income to total property value (Net Operating Income)
NIR	Net Income Ratio (also known as Net Operating Income Ratio or 1 - OER)
NLA	Net Leasable Area
NOI	Net Operating Income
OER	Operating Expense Ratio (or 1 - NIR)
PAV	Property Assessment Valuation (published 1996)
PGI	Potential Gross Income
PW	Present Worth
R_B	Building Capitalization Rate (also known as the Improvement Capitalization Rate)
R_E	Equity Capitalization Rate (also known as the Equity Dividend Rate or Cash on Cash Rate)
R_L	Land Capitalization Rate
R_M	Mortgage Capitalization Rate
R_O	Overall Capitalization Rate (also known as OAR)
V_B	Value of the Building (Improvement)
V_E	Value of Equity
VIF	Value Income & Factor formula
V_L	Value of the Land
V_M	Value of the Mortgage (Mortgage Principal)
V&C	Vacancy & Collection Loss
Y_B	Building Yield Rate
Y_E	Equity Yield Rate
Y_L	Land Yield Rate
Y_M	Mortgage Yield Rate (Mortgage Interest Rate or lender's yield on the loan)
Y_O	Overall Yield Rate (also known as the Discount Rate or "r")

Appendix II - Compound Interest Tables

Compound Interest Tables

Annual Compound Interest tables follow:

6% Annual Compound Interest Table

Period	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.060000	1.000000	1.000000	0.943396	0.943396	1.06000000
2	1.123600	2.060000	0.485437	0.889996	1.833393	0.54543689
3	1.191016	3.183600	0.314110	0.839619	2.673012	0.37410981
4	1.262477	4.374616	0.228591	0.792094	3.465106	0.28859149
5	1.338226	5.637093	0.177396	0.747258	4.212364	0.23739640
6	1.418519	6.975319	0.143363	0.704961	4.917324	0.20336263
7	1.503630	8.393838	0.119135	0.665057	5.582381	0.17913502
8	1.593848	9.897468	0.101036	0.627412	6.209794	0.16103594
9	1.689479	11.491316	0.087022	0.591898	6.801692	0.14702224
10	1.790848	13.180795	0.075868	0.558395	7.360087	0.13586796
11	1.898299	14.971643	0.066793	0.526788	7.886875	0.12679294
12	2.012196	16.869941	0.059277	0.496969	8.383844	0.11927703
13	2.132928	18.882138	0.052960	0.468839	8.852683	0.11296011
14	2.260904	21.015066	0.047585	0.442301	9.294984	0.10758491
15	2.396558	23.275970	0.042963	0.417265	9.712249	0.10296276
16	2.540352	25.672528	0.038952	0.393646	10.105895	0.09895214
17	2.692773	28.212880	0.035445	0.371364	10.477260	0.09544480
18	2.854339	30.905653	0.032357	0.350344	10.827603	0.09235654
19	3.025600	33.759992	0.029621	0.330513	11.158116	0.08962086
20	3.207135	36.785591	0.027185	0.311805	11.469921	0.08718456
21	3.399564	39.992727	0.025005	0.294155	11.764077	0.08500455
22	3.603537	43.392290	0.023046	0.277505	12.041582	0.08304557
23	3.819750	46.995828	0.021278	0.261797	12.303379	0.08127848
24	4.048935	50.815577	0.019679	0.246979	12.550358	0.07967900
25	4.291871	54.864512	0.018227	0.232999	12.783356	0.07822672
26	4.549383	59.156383	0.016904	0.219810	13.003166	0.07690435
27	4.822346	63.705766	0.015697	0.207368	13.210534	0.07569717
28	5.111687	68.528112	0.014593	0.195630	13.406164	0.07459255
29	5.418388	73.639798	0.013580	0.184557	13.590721	0.07357961
30	5.743491	79.058186	0.012649	0.174110	13.764831	0.07264891
31	6.088101	84.801677	0.011792	0.164255	13.929086	0.07179222
32	6.453387	90.889778	0.011002	0.154957	14.084043	0.07100234
33	6.840590	97.343165	0.010273	0.146186	14.230230	0.07027293
34	7.251025	104.183755	0.009598	0.137912	14.368141	0.06959843
35	7.686087	111.434780	0.008974	0.130105	14.498246	0.06897386
36	8.147252	119.120867	0.008395	0.122741	14.620987	0.06839483
37	8.636087	127.268119	0.007857	0.115793	14.736780	0.06785743
38	9.154252	135.904206	0.007358	0.109239	14.846019	0.06735812
39	9.703507	145.058458	0.006894	0.103056	14.949075	0.06689377
40	10.285718	154.761966	0.006462	0.097222	15.046297	0.06646154

6% Monthly Compound Interest Table

Month	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.005000	1.000000	1.000000	0.995025	0.995025	1.00500000
2	1.010025	2.005000	0.498753	0.990075	1.985099	0.50375312
3	1.015075	3.015025	0.331672	0.985149	2.970248	0.33667221
4	1.020151	4.030100	0.248133	0.980248	3.950496	0.25313279
5	1.025251	5.050251	0.198010	0.975371	4.925866	0.20300997
6	1.030378	6.075502	0.164595	0.970518	5.896384	0.16959546
7	1.035529	7.105879	0.140729	0.965690	6.862074	0.14572854
8	1.040707	8.141409	0.122829	0.960885	7.822959	0.12782886
9	1.045911	9.182116	0.108907	0.956105	8.779064	0.11390736
10	1.051140	10.228026	0.097771	0.951348	9.730412	0.10277057
11	1.056396	11.279167	0.088659	0.946615	10.677027	0.09365903
12	1.061678	12.335562	0.081066	0.941905	11.618932	0.08606643
Year						
1	1.061678	12.335562	0.081066	0.941905	11.618932	0.08606643
2	1.127160	25.431955	0.039321	0.887186	22.562866	0.04432061
3	1.196681	39.336105	0.025422	0.835645	32.871016	0.03042194
4	1.270489	54.097832	0.018485	0.787098	42.580318	0.02348503
5	1.348850	69.770031	0.014333	0.741372	51.725561	0.01933280
6	1.432044	86.408856	0.011573	0.698302	60.339514	0.01657289
7	1.520370	104.073927	0.009609	0.657735	68.453042	0.01460855
8	1.614143	122.828542	0.008141	0.619524	76.095218	0.01314143
9	1.713699	142.739900	0.007006	0.583533	83.293424	0.01200575
10	1.819397	163.879347	0.006102	0.549633	90.073453	0.01110205
11	1.931613	186.322629	0.005367	0.517702	96.459599	0.01036703
12	2.050751	210.150163	0.004759	0.487626	102.474743	0.00975850
13	2.177237	235.447328	0.004247	0.459298	108.140440	0.00924723
14	2.311524	262.304766	0.003812	0.432615	113.476990	0.00881236
15	2.454094	290.818712	0.003439	0.407482	118.503515	0.00843857
16	2.605457	321.091337	0.003114	0.383810	123.238025	0.00811438
17	2.766156	353.231110	0.002831	0.361513	127.697486	0.00783101
18	2.936766	387.353194	0.002582	0.340511	131.897876	0.00758162
19	3.117899	423.579854	0.002361	0.320729	135.854246	0.00736083
20	3.310204	462.040895	0.002164	0.302096	139.580772	0.00716431
21	3.514371	502.874129	0.001989	0.284546	143.090806	0.00698857
22	3.731129	546.225867	0.001831	0.268015	146.396927	0.00683074
23	3.961257	592.251446	0.001688	0.252445	149.510979	0.00668847
24	4.205579	641.115782	0.001560	0.237779	152.444121	0.00655978
25	4.464970	692.993962	0.001443	0.223966	155.206864	0.00644301
30	6.022575	1004.515042	0.000996	0.166042	166.791614	0.00599551
35	8.123551	1424.710299	0.000702	0.123099	175.380226	0.00570190
40	10.957454	1991.490734	0.000502	0.091262	181.747584	0.00550214

7% Annual Compound Interest Table

Period	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.070000	1.000000	1.000000	0.934579	0.934579	1.07000000
2	1.144900	2.070000	0.483092	0.873439	1.808018	0.55309179
3	1.225043	3.214900	0.311052	0.816298	2.624316	0.38105167
4	1.310796	4.439943	0.225228	0.762895	3.387211	0.29522812
5	1.402552	5.750739	0.173891	0.712986	4.100197	0.24389069
6	1.500730	7.153291	0.139796	0.666342	4.766540	0.20979580
7	1.605781	8.654021	0.115553	0.622750	5.389289	0.18555322
8	1.718186	10.259803	0.097468	0.582009	5.971299	0.16746776
9	1.838459	11.977989	0.083486	0.543934	6.515232	0.15348647
10	1.967151	13.816448	0.072378	0.508349	7.023582	0.14237750
11	2.104852	15.783599	0.063357	0.475093	7.498674	0.13335690
12	2.252192	17.888451	0.055902	0.444012	7.942686	0.12590199
13	2.409845	20.140643	0.049651	0.414964	8.357651	0.11965085
14	2.578534	22.550488	0.044345	0.387817	8.745468	0.11434494
15	2.759032	25.129022	0.039795	0.362446	9.107914	0.10979462
16	2.952164	27.888054	0.035858	0.338735	9.446649	0.10585765
17	3.158815	30.840217	0.032425	0.316574	9.763223	0.10242519
18	3.379932	33.999033	0.029413	0.295864	10.059087	0.09941260
19	3.616528	37.378965	0.026753	0.276508	10.335595	0.09675301
20	3.869684	40.995492	0.024393	0.258419	10.594014	0.09439293
21	4.140562	44.865177	0.022289	0.241513	10.835527	0.09228900
22	4.430402	49.005739	0.020406	0.225713	11.061240	0.09040577
23	4.740530	53.436141	0.018714	0.210947	11.272187	0.08871393
24	5.072367	58.176671	0.017189	0.197147	11.469334	0.08718902
25	5.427433	63.249038	0.015811	0.184249	11.653583	0.08581052
26	5.807353	68.676470	0.014561	0.172195	11.825779	0.08456103
27	6.213868	74.483823	0.013426	0.160930	11.986709	0.08342573
28	6.648838	80.697691	0.012392	0.150402	12.137111	0.08239193
29	7.114257	87.346529	0.011449	0.140563	12.277674	0.08144865
30	7.612255	94.460786	0.010586	0.131367	12.409041	0.08058640
31	8.145113	102.073041	0.009797	0.122773	12.531814	0.07979691
32	8.715271	110.218154	0.009073	0.114741	12.646555	0.07907292
33	9.325340	118.933425	0.008408	0.107235	12.753790	0.07840807
34	9.978114	128.258765	0.007797	0.100219	12.854009	0.07779674
35	10.676581	138.236878	0.007234	0.093663	12.947672	0.07723396
36	11.423942	148.913460	0.006715	0.087535	13.035208	0.07671531
37	12.223618	160.337402	0.006237	0.081809	13.117017	0.07623685
38	13.079271	172.561020	0.005795	0.076457	13.193473	0.07579505
39	13.994820	185.640292	0.005387	0.071455	13.264928	0.07538676
40	14.974458	199.635112	0.005009	0.066780	13.331709	0.07500914

7% Monthly Compound Interest Table

Month	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.005833	1.000000	1.000000	0.994200	0.994200	1.00583333
2	1.011701	2.005833	0.498546	0.988435	1.982635	0.50437924
3	1.017602	3.017534	0.331396	0.982702	2.965337	0.33722976
4	1.023538	4.035136	0.247823	0.977003	3.942340	0.25365644
5	1.029509	5.058675	0.197680	0.971337	4.913677	0.20351357
6	1.035514	6.088184	0.164253	0.965704	5.879381	0.17008594
7	1.041555	7.123698	0.140377	0.960103	6.839484	0.14620986
8	1.047631	8.165253	0.122470	0.954535	7.794019	0.12830352
9	1.053742	9.212883	0.108544	0.948999	8.743018	0.11437698
10	1.059889	10.266625	0.097403	0.943495	9.686513	0.10323632
11	1.066071	11.326514	0.088288	0.938024	10.624537	0.09412175
12	1.072290	12.392585	0.080693	0.932583	11.557120	0.08652675
Year						
1	1.072290	12.392585	0.080693	0.932583	11.557120	0.08652675
2	1.149806	25.681032	0.038939	0.869712	22.335099	0.04477258
3	1.232926	39.930101	0.025044	0.811079	32.386464	0.03087710
4	1.322054	55.209236	0.018113	0.756399	41.760201	0.02394624
5	1.417625	71.592902	0.013968	0.705405	50.501994	0.01980120
6	1.520106	89.160944	0.011216	0.657849	58.654444	0.01704901
7	1.629994	107.998981	0.009259	0.613499	66.257285	0.01509268
8	1.747826	128.198821	0.007800	0.572139	73.347569	0.01363372
9	1.874177	149.858909	0.006673	0.533568	79.959850	0.01250628
10	2.009661	173.084807	0.005778	0.497596	86.126354	0.01161085
11	2.154940	197.989707	0.005051	0.464050	91.877134	0.01088410
12	2.310721	224.694985	0.004450	0.432765	97.240216	0.01028381
13	2.477763	253.330789	0.003947	0.403590	102.241738	0.00978074
14	2.656881	284.036677	0.003521	0.376381	106.906074	0.00935401
15	2.848947	316.962297	0.003155	0.351007	111.255958	0.00898828
16	3.054897	352.268112	0.002839	0.327343	115.312587	0.00867208
17	3.275736	390.126188	0.002563	0.305275	119.095732	0.00839661
18	3.512539	430.721027	0.002322	0.284694	122.623831	0.00815502
19	3.766461	474.250470	0.002109	0.265501	125.914077	0.00794192
20	4.038739	520.926660	0.001920	0.247602	128.982506	0.00775299
21	4.330700	570.977075	0.001751	0.230910	131.844073	0.00758472
22	4.643766	624.645640	0.001601	0.215342	134.512723	0.00743424
23	4.979464	682.193909	0.001466	0.200825	137.001461	0.00729919
24	5.339430	743.902347	0.001344	0.187286	139.322418	0.00717760
25	5.725418	810.071693	0.001234	0.174660	141.486903	0.00706779
30	8.116497	1219.970996	0.000820	0.123206	150.307568	0.00665302
35	11.506152	1801.054601	0.000555	0.086910	156.529709	0.00638856
40	16.311411	2624.813398	0.000381	0.061307	160.918839	0.00621431

8% Annual Compound Interest Table

Period	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.080000	1.000000	1.000000	0.925926	0.925926	1.08000000
2	1.166400	2.080000	0.480769	0.857339	1.783265	0.56076923
3	1.259712	3.246400	0.308034	0.793832	2.577097	0.38803351
4	1.360489	4.506112	0.221921	0.735030	3.312127	0.30192080
5	1.469328	5.866601	0.170456	0.680583	3.992710	0.25045645
6	1.586874	7.335929	0.136315	0.630170	4.622880	0.21631539
7	1.713824	8.922803	0.112072	0.583490	5.206370	0.19207240
8	1.850930	10.636628	0.094015	0.540269	5.746639	0.17401476
9	1.999005	12.487558	0.080080	0.500249	6.246888	0.16007971
10	2.158925	14.486562	0.069029	0.463193	6.710081	0.14902949
11	2.331639	16.645487	0.060076	0.428883	7.138964	0.14007634
12	2.518170	18.977126	0.052695	0.397114	7.536078	0.13269502
13	2.719624	21.495297	0.046522	0.367698	7.903776	0.12652181
14	2.937194	24.214920	0.041297	0.340461	8.244237	0.12129685
15	3.172169	27.152114	0.036830	0.315242	8.559479	0.11682954
16	3.425943	30.324283	0.032977	0.291890	8.851369	0.11297687
17	3.700018	33.750226	0.029629	0.270269	9.121638	0.10962943
18	3.996019	37.450244	0.026702	0.250249	9.371887	0.10670210
19	4.315701	41.446263	0.024128	0.231712	9.603599	0.10412763
20	4.660957	45.761964	0.021852	0.214548	9.818147	0.10185221
21	5.033834	50.422921	0.019832	0.198656	10.016803	0.09983225
22	5.436540	55.456755	0.018032	0.183941	10.200744	0.09803207
23	5.871464	60.893296	0.016422	0.170315	10.371059	0.09642217
24	6.341181	66.764759	0.014978	0.157699	10.528758	0.09497796
25	6.848475	73.105940	0.013679	0.146018	10.674776	0.09367878
26	7.396353	79.954415	0.012507	0.135202	10.809978	0.09250713
27	7.988061	87.350768	0.011448	0.125187	10.935165	0.09144810
28	8.627106	95.338830	0.010489	0.115914	11.051078	0.09048891
29	9.317275	103.965936	0.009619	0.107328	11.158406	0.08961854
30	10.062657	113.283211	0.008827	0.099377	11.257783	0.08882743
31	10.867669	123.345868	0.008107	0.092016	11.349799	0.08810728
32	11.737083	134.213537	0.007451	0.085200	11.434999	0.08745081
33	12.676050	145.950620	0.006852	0.078889	11.513888	0.08685163
34	13.690134	158.626670	0.006304	0.073045	11.586934	0.08630411
35	14.785344	172.316804	0.005803	0.067635	11.654568	0.08580326
36	15.968172	187.102148	0.005345	0.062625	11.717193	0.08534467
37	17.245626	203.070320	0.004924	0.057986	11.775179	0.08492440
38	18.625276	220.315945	0.004539	0.053690	11.828869	0.08453894
39	20.115298	238.941221	0.004185	0.049713	11.878582	0.08418513
40	21.724521	259.056519	0.003860	0.046031	11.924613	0.08386016

8% Monthly Compound Interest Table

Month	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.006667	1.000000	1.000000	0.993377	0.993377	1.00666667
2	1.013378	2.006667	0.498339	0.986799	1.980176	0.50500554
3	1.020134	3.020044	0.331121	0.980264	2.960440	0.33778762
4	1.026935	4.040178	0.247514	0.973772	3.934212	0.25418051
5	1.033781	5.067113	0.197351	0.967323	4.901535	0.20401772
6	1.040673	6.100893	0.163910	0.960917	5.862452	0.17057709
7	1.047610	7.141566	0.140025	0.954553	6.817005	0.14669198
8	1.054595	8.189176	0.122112	0.948232	7.765237	0.12877907
9	1.061625	9.243771	0.108181	0.941952	8.707189	0.11484763
10	1.068703	10.305396	0.097037	0.935714	9.642903	0.10370321
11	1.075827	11.374099	0.087919	0.929517	10.572420	0.09458572
12	1.083000	12.449926	0.080322	0.923361	11.495782	0.08698843
Year						
1	1.083000	12.449926	0.080322	0.923361	11.495782	0.08698843
2	1.172888	25.933190	0.038561	0.852596	22.110544	0.04522729
3	1.270237	40.535558	0.024670	0.787255	31.911806	0.03133637
4	1.375666	56.349915	0.017746	0.726921	40.961913	0.02441292
5	1.489846	73.476856	0.013610	0.671210	49.318433	0.02027639
6	1.613502	92.025325	0.010867	0.619770	57.034522	0.01753324
7	1.747422	112.113308	0.008920	0.572272	64.159261	0.01558621
8	1.892457	133.868583	0.007470	0.528414	70.737970	0.01413668
9	2.049530	157.429535	0.006352	0.487917	76.812497	0.01301871
10	2.219640	182.946035	0.005466	0.450523	82.421481	0.01213276
11	2.403869	210.580392	0.004749	0.415996	87.600600	0.01141545
12	2.603389	240.508387	0.004158	0.384115	92.382800	0.01082453
13	2.819469	272.920390	0.003664	0.354677	96.798498	0.01033074
14	3.053484	308.022574	0.003247	0.327495	100.875784	0.00991318
15	3.306921	346.038222	0.002890	0.302396	104.640592	0.00955652
16	3.581394	387.209149	0.002583	0.279221	108.116871	0.00924925
17	3.878648	431.797244	0.002316	0.257822	111.326733	0.00898257
18	4.200574	480.086128	0.002083	0.238063	114.290596	0.00874963
19	4.549220	532.382966	0.001878	0.219818	117.027313	0.00854501
20	4.926803	589.020416	0.001698	0.202971	119.554292	0.00836440
21	5.335725	650.358746	0.001538	0.187416	121.887606	0.00820428
22	5.778588	716.788127	0.001395	0.173053	124.042099	0.00806178
23	6.258207	788.731114	0.001268	0.159790	126.031475	0.00793453
24	6.777636	866.645333	0.001154	0.147544	127.868388	0.00782054
25	7.340176	951.026395	0.001051	0.136237	129.564523	0.00771816
30	10.935730	1490.359449	0.000671	0.091443	136.283494	0.00733765
35	16.292550	2293.882485	0.000436	0.061378	140.793338	0.00710261
40	24.273386	3491.007831	0.000286	0.041197	143.820392	0.00695312

10% Annual Compound Interest Table

Period	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.100000	1.000000	1.000000	0.909091	0.909091	1.10000000
2	1.210000	2.100000	0.476190	0.826446	1.735537	0.57619048
3	1.331000	3.310000	0.302115	0.751315	2.486852	0.40211480
4	1.464100	4.641000	0.215471	0.683013	3.169865	0.31547080
5	1.610510	6.105100	0.163797	0.620921	3.790787	0.26379748
6	1.771561	7.715610	0.129607	0.564474	4.355261	0.22960738
7	1.948717	9.487171	0.105405	0.513158	4.868419	0.20540550
8	2.143589	11.435888	0.087444	0.466507	5.334926	0.18744402
9	2.357948	13.579477	0.073641	0.424098	5.759024	0.17364054
10	2.593742	15.937425	0.062745	0.385543	6.144567	0.16274539
11	2.853117	18.531167	0.053963	0.350494	6.495061	0.15396314
12	3.138428	21.384284	0.046763	0.318631	6.813692	0.14676332
13	3.452271	24.522712	0.040779	0.289664	7.103356	0.14077852
14	3.797498	27.974983	0.035746	0.263331	7.366687	0.13574622
15	4.177248	31.772482	0.031474	0.239392	7.606080	0.13147378
16	4.594973	35.949730	0.027817	0.217629	7.823709	0.12781662
17	5.054470	40.544703	0.024664	0.197845	8.021553	0.12466413
18	5.559917	45.599173	0.021930	0.179859	8.201412	0.12193022
19	6.115909	51.159090	0.019547	0.163508	8.364920	0.11954687
20	6.727500	57.274999	0.017460	0.148644	8.513564	0.11745962
21	7.400250	64.002499	0.015624	0.135131	8.648694	0.11562439
22	8.140275	71.402749	0.014005	0.122846	8.771540	0.11400506
23	8.954302	79.543024	0.012572	0.111678	8.883218	0.11257181
24	9.849733	88.497327	0.011300	0.101526	8.984744	0.11129978
25	10.834706	98.347059	0.010168	0.092296	9.077040	0.11016807
26	11.918177	109.181765	0.009159	0.083905	9.160945	0.10915904
27	13.109994	121.099942	0.008258	0.076278	9.237223	0.10825764
28	14.420994	134.209936	0.007451	0.069343	9.306567	0.10745101
29	15.863093	148.630930	0.006728	0.063039	9.369606	0.10672807
30	17.449402	164.494023	0.006079	0.057309	9.426914	0.10607925
31	19.194342	181.943425	0.005496	0.052099	9.479013	0.10549621
32	21.113777	201.137767	0.004972	0.047362	9.526376	0.10497172
33	23.225154	222.251544	0.004499	0.043057	9.569432	0.10449941
34	25.547670	245.476699	0.004074	0.039143	9.608575	0.10407371
35	28.102437	271.024368	0.003690	0.035584	9.644159	0.10368971
36	30.912681	299.126805	0.003343	0.032349	9.676508	0.10334306
37	34.003949	330.039486	0.003030	0.029408	9.705917	0.10302994
38	37.404343	364.043434	0.002747	0.026735	9.732651	0.10274692
39	41.144778	401.447778	0.002491	0.024304	9.756956	0.10249098
40	45.259256	442.592556	0.002259	0.022095	9.779051	0.10225941

10% Monthly Compound Interest Table

Month	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.008333	1.000000	1.000000	0.991736	0.991736	1.00833333
2	1.016736	2.008333	0.497925	0.983539	1.975275	0.50625864
3	1.025209	3.025069	0.330571	0.975411	2.950686	0.33890426
4	1.033752	4.050278	0.246897	0.967350	3.918036	0.25522994
5	1.042367	5.084031	0.196694	0.959355	4.877391	0.20502766
6	1.051053	6.126398	0.163228	0.951427	5.828817	0.17156139
7	1.059812	7.177451	0.139325	0.943563	6.772381	0.14765856
8	1.068644	8.237263	0.121400	0.935765	7.708146	0.12973288
9	1.077549	9.305907	0.107459	0.928032	8.636178	0.11579196
10	1.086529	10.383456	0.096307	0.920362	9.556540	0.10464038
11	1.095583	11.469985	0.087184	0.912756	10.469296	0.09551741
12	1.104713	12.565568	0.079583	0.905212	11.374508	0.08791589
Year						
1	1.104713	12.565568	0.079583	0.905212	11.374508	0.08791589
2	1.220391	26.446915	0.037812	0.819410	21.670855	0.04614493
3	1.348182	41.781821	0.023934	0.741740	30.991236	0.03226719
4	1.489354	58.722492	0.017029	0.671432	39.428160	0.02536258
5	1.645309	77.437072	0.012914	0.607789	47.065369	0.02124704
6	1.817594	98.111314	0.010193	0.550178	53.978665	0.01852584
7	2.007920	120.950418	0.008268	0.498028	60.236667	0.01660118
8	2.218176	146.181076	0.006841	0.450821	65.901488	0.01517416
9	2.450448	174.053713	0.005745	0.408089	71.029355	0.01407869
10	2.707041	204.844979	0.004882	0.369407	75.671163	0.01321507
11	2.990504	238.860493	0.004187	0.334392	79.872986	0.01251988
12	3.303649	276.437876	0.003617	0.302696	83.676528	0.01195078
13	3.649584	317.950102	0.003145	0.274004	87.119542	0.01147848
14	4.031743	363.809201	0.002749	0.248032	90.236201	0.01108203
15	4.453920	414.470346	0.002413	0.224521	93.057439	0.01074605
16	4.920303	470.436376	0.002126	0.203240	95.611259	0.01045902
17	5.435523	532.262780	0.001879	0.183975	97.923008	0.01021210
18	6.004693	600.563216	0.001665	0.166536	100.015633	0.00999844
19	6.633463	676.015601	0.001479	0.150751	101.909902	0.00981259
20	7.328074	759.368836	0.001317	0.136462	103.624619	0.00965022
21	8.095419	851.450244	0.001174	0.123527	105.176801	0.00950780
22	8.943115	953.173779	0.001049	0.111818	106.581856	0.00938246
23	9.879576	1065.549097	0.000938	0.101219	107.853730	0.00927182
24	10.914097	1189.691580	0.000841	0.091625	109.005045	0.00917389
25	12.056945	1326.833403	0.000754	0.082940	110.047230	0.00908701
30	19.837399	2260.487925	0.000442	0.050410	113.950820	0.00877572
35	32.638650	3796.638052	0.000263	0.030639	116.323377	0.00859672
40	53.700663	6324.079581	0.000158	0.018622	117.765391	0.00849146

12% Annual Compound Interest Table

Period	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.120000	1.000000	1.000000	0.892857	0.892857	1.12000000
2	1.254400	2.120000	0.471698	0.797194	1.690051	0.59169811
3	1.404928	3.374400	0.296349	0.711780	2.401831	0.41634898
4	1.573519	4.779328	0.209234	0.635518	3.037349	0.32923444
5	1.762342	6.352847	0.157410	0.567427	3.604776	0.27740973
6	1.973823	8.115189	0.123226	0.506631	4.111407	0.24322572
7	2.210681	10.089012	0.099118	0.452349	4.563757	0.21911774
8	2.475963	12.299693	0.081303	0.403883	4.967640	0.20130284
9	2.773079	14.775656	0.067679	0.360610	5.328250	0.18767889
10	3.105848	17.548735	0.056984	0.321973	5.650223	0.17698416
11	3.478550	20.654583	0.048415	0.287476	5.937699	0.16841540
12	3.895976	24.133133	0.041437	0.256675	6.194374	0.16143681
13	4.363493	28.029109	0.035677	0.229174	6.423548	0.15567720
14	4.887112	32.392602	0.030871	0.204620	6.628168	0.15087125
15	5.473566	37.279715	0.026824	0.182696	6.810864	0.14682424
16	6.130394	42.753280	0.023390	0.163122	6.973986	0.14339002
17	6.866041	48.883674	0.020457	0.145644	7.119630	0.14045673
18	7.689966	55.749715	0.017937	0.130040	7.249670	0.13793731
19	8.612762	63.439681	0.015763	0.116107	7.365777	0.13576300
20	9.646293	72.052442	0.013879	0.103667	7.469444	0.13387878
21	10.803848	81.698736	0.012240	0.092560	7.562003	0.13224009
22	12.100310	92.502584	0.010811	0.082643	7.644646	0.13081051
23	13.552347	104.602894	0.009560	0.073788	7.718434	0.12955996
24	15.178629	118.155241	0.008463	0.065882	7.784316	0.12846344
25	17.000064	133.333870	0.007500	0.058823	7.843139	0.12749997
26	19.040072	150.333934	0.006652	0.052521	7.895660	0.12665186
27	21.324881	169.374007	0.005904	0.046894	7.942554	0.12590409
28	23.883866	190.698887	0.005244	0.041869	7.984423	0.12524387
29	26.749930	214.582754	0.004660	0.037383	8.021806	0.12466021
30	29.959922	241.332684	0.004144	0.033378	8.055184	0.12414366
31	33.555113	271.292606	0.003686	0.029802	8.084986	0.12368606
32	37.581726	304.847719	0.003280	0.026609	8.111594	0.12328033
33	42.091533	342.429446	0.002920	0.023758	8.135352	0.12292031
34	47.142517	384.520979	0.002601	0.021212	8.156564	0.12260064
35	52.799620	431.663496	0.002317	0.018940	8.175504	0.12231662
36	59.135574	484.463116	0.002064	0.016910	8.192414	0.12206414
37	66.231843	543.598690	0.001840	0.015098	8.207513	0.12183959
38	74.179664	609.830533	0.001640	0.013481	8.220993	0.12163980
39	83.081224	684.010197	0.001462	0.012036	8.233030	0.12146197
40	93.050970	767.091420	0.001304	0.010747	8.243777	0.12130363

12% Monthly Compound Interest Table

Month	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.010000	1.000000	1.000000	0.990099	0.990099	1.01000000
2	1.020100	2.010000	0.497512	0.980296	1.970395	0.50751244
3	1.030301	3.030100	0.330022	0.970590	2.940985	0.34002211
4	1.040604	4.060401	0.246281	0.960980	3.901966	0.25628109
5	1.051010	5.101005	0.196040	0.951466	4.853431	0.20603980
6	1.061520	6.152015	0.162548	0.942045	5.795476	0.17254837
7	1.072135	7.213535	0.138628	0.932718	6.728195	0.14862828
8	1.082857	8.285671	0.120690	0.923483	7.651678	0.13069029
9	1.093685	9.368527	0.106740	0.914340	8.566018	0.11674036
10	1.104622	10.462213	0.095582	0.905287	9.471305	0.10558208
11	1.115668	11.566835	0.086454	0.896324	10.367628	0.09645408
12	1.126825	12.682503	0.078849	0.887449	11.255077	0.08884879
Year						
1	1.126825	12.682503	0.078849	0.887449	11.255077	0.08884879
2	1.269735	26.973465	0.037073	0.787566	21.243387	0.04707347
3	1.430769	43.076878	0.023214	0.698925	30.107505	0.03321431
4	1.612226	61.222608	0.016334	0.620260	37.973959	0.02633384
5	1.816697	81.669670	0.012244	0.550450	44.955038	0.02224445
6	2.047099	104.709931	0.009550	0.488496	51.150391	0.01955019
7	2.306723	130.672274	0.007653	0.433515	56.648453	0.01765273
8	2.599273	159.927293	0.006253	0.384723	61.527703	0.01625284
9	2.928926	192.892579	0.005184	0.341422	65.857790	0.01518423
10	3.300387	230.038689	0.004347	0.302995	69.700522	0.01434709
11	3.718959	271.895856	0.003678	0.268892	73.110752	0.01367788
12	4.190616	319.061559	0.003134	0.238628	76.137157	0.01313419
13	4.722091	372.209054	0.002687	0.211771	78.822939	0.01268666
14	5.320970	432.096982	0.002314	0.187936	81.206434	0.01231430
15	5.995802	499.580198	0.002002	0.166783	83.321664	0.01200168
16	6.756220	575.621974	0.001737	0.148012	85.198824	0.01173725
17	7.613078	661.307751	0.001512	0.131353	86.864707	0.01151216
18	8.578606	757.860630	0.001320	0.116569	88.343095	0.01131950
19	9.666588	866.658830	0.001154	0.103449	89.655089	0.01115386
20	10.892554	989.255365	0.001011	0.091806	90.819416	0.01101086
21	12.274002	1127.400210	0.000887	0.081473	91.852698	0.01088700
22	13.830653	1283.065279	0.000779	0.072303	92.769683	0.01077938
23	15.584726	1458.472574	0.000686	0.064165	93.583461	0.01068565
24	17.561259	1656.125905	0.000604	0.056944	94.305647	0.01060382
25	19.788466	1878.846626	0.000532	0.050534	94.946551	0.01053224
30	35.949641	3494.964133	0.000286	0.027817	97.218331	0.01028613
35	65.309595	6430.959471	0.000155	0.015312	98.468831	0.01015550
40	118.647725	11764.772510	0.000085	0.008428	99.157169	0.01008500