GEORGIA DEPARTMENT OF REVENUE

LOCAL GOVERNMENT SERVICES DIVISION

Course II

The Income Approach to Value

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Acknowledgement is given to IAAO.org for materials contained herein.

Chapter 1

Objectives

Upon completion of this chapter, you will be able to:

- ✓ Recognize the definition of income-producing property.
- ✓ Recognize the difference between market value and investment value and know why these differences are significant in deriving value estimates by the income approach.
- ✓ Explain how the appraiser's understanding of anticipation, change, and the other basic appraisal principles aids in the analysis of market data necessary in the application of the income approach.
- ✓ Recognize the different investor requirements for income-producing properties.
- ✓ Recognize the factors that influence the behavior of investors in real estate.
- ✓ Identify and explain the various types of financing available to investors in real estate.
- ✓ List the sources of financing for purchasing real estate.
- ✓ Identify and be able to explain the following components of capitalization rates: discount rate; recapture rate; and effective tax rate.
- ✓ Understand the basic concepts of the income approach model and how supply and demand factors interact in specific markets.
- ✓ Understand the income capitalization formulas (IRV, which is Income equals Rate times Value and VIM which is Value equals Income times the Factor) in appraising income-producing properties.

Introduction

This course is designed to provide participants with an understanding of the capitalization theory, methods, and techniques used in the appraisal of income producing properties. It will provide the participant with a working knowledge of appraisal theory, assessment procedures and income appraisal techniques necessary to estimate the market value of vacant and improved property. Chapter 1 focuses on the principles and theory underlying the income capitalization approach to estimating market value.

I. Introduction to Income Capitalization Approach

A. The income capitalization approach is one of the three approaches to value. In this approach, the value of an income-producing property is estimated by converting anticipated benefits (income or rent) by the owner of the income producing property.

B. Income capitalization is based upon the economic principles of:

1. Anticipation – value is created by the expectation of benefits to be derived in the future.

2. Change – investor's expectations of changes in income levels, the expenses required to ensure income, and probable increases or decreases in property must be addressed and forecast.

3. Competition – competition means that an excess of one type of facility will decrease the value of all such facilities. Excess competition destroys balance. Competition is created by the potential for profits, which attract new buyers and sellers to the market. Competition among buyers may lead to shortages, which increases prices and profits to sellers.

4. Substitution – the prices, rents, and rates of return of property tend to be set by the current prices, rents, and rates of return for equally desirable properties. The principle of substitution is market-oriented and

provides the bases for estimating rents and expenses and selecting the proper discount rate or capitalization rate for the subject property.

5. Balance – a suitable balance among types and locations of income-producing properties affects value; an imbalance in use may result in declining value.

6. Contribution – the value of a component of real estate can be measured by the amount it contributes to net operating income because net operating income can be capitalized into value.

7. Supply – the amount of product that producers are willing to sell under various conditions during a given period.

8. Demand – the amount of product buyers are willing and able to purchase during some period, given the choices available to them.

C. 48-5-2(3) Fair market value of property means the amount a knowledgeable buyer would pay for the property and a willing seller would accept for the property at an arm's length, bona fide sale. The income approach, if data is available, shall be utilized in determining the fair market value of income-producing property, and, if actual income and expense data are voluntarily supplied by the property owner, such data shall be considered in such determination.

In the US, the most common definition of Market Value is the one promulgated for use in Federally regulated residential mortgage financing: The most probable price which a property should bring in a competitive and open market under all conditions requisite to a fair sale, the buyer and seller, each acting prudently, knowledgeably and assuming the price is not affected by undue stimulus. Implicit in this definition is the consummation of a sale as of a specified date and the passing of title from seller to buyer under conditions whereby: (1) buyer and

seller are typically motivated; (2) both parties are well informed or well advised, and each acting in what he or she considers his or her own best interest; (3) a reasonable time is allowed for exposure in the open market; (4) payment is made in terms of cash in U. S. dollars or in terms of financial arrangements comparable thereto; and (5) the price represents the normal consideration for the property sold unaffected by special or creative financing or sales concessions granted by anyone associated with the sale.

1. Buyer and seller typical motivated;

2. Both parties are well informed and acting in their own best interest;

3. A reasonable time is allowed for exposure in the open market;

4. Payment is made in terms of cash in U.S. dollars or in terms of financial arrangements comparable thereto; and

5. The price represents the normal consideration for the property sold unaffected by special or creative financing or sales concessions granted by anyone associated with the sale.

D. Definition of investment value – The worth of an investment property to a particular investor. Investment value may or may not coincide with market value depending on the requirements of the specific investor.

E. Review components of property rights

Ownership entity – individual, corporate shareholders, partnership interests
Financial interest – equity, debt (mortgage)
Legal estate – fee simple, leased fee, leasehold

Real Estate Investments

A. Real estate competes with other investments for the dollars of the investor.

B. An investor analyzes the various opportunities available and asks the following questions:

1. How much will it cost?

2. How much will I get back?

3. When will I get it back?

4. What are the risks?

5. What is the return of a real estate investment compared to other investments with similar risks?

C. The objectives of the investor vary.

1. All desire a return **on** their investment.

2. All desire a return **of** their investment.

3. Investors are different.

   a. Some require a substantial annual return on investment.
   b. Some are interested only in potential growth of the investment.
   c. Some are interested in both a substantial return and potential growth.

D. Factors influencing the behavior of investors are diverse. To better understand these factors; compare an investment in a savings account and an investment in real estate.

1. **Safety** of the investment – Investors are adverse to risk. A savings account insured by the government is relatively free of risk. Real estate may be a risky investment due to a variety of factors affecting its economic environment.

2. **Liquidity** of the investment – A savings account is accessible on demand for conversion to cash. Real Estate investment requires time to convert to cash. Real estate requires time to secure a contract and sell. This may take months before title transfers and the seller is paid.

3. **Size** of the investment – A savings account can be opened with a very small amount of money. Real estate generally requires a substantial commitment of capital.

4. **Use as collateral** – The entire amount in a savings account can be used as collateral for a loan. Lending institutions normally will only allow a percentage of the real estate's market value to be used as collateral.

5. **Holding period** of the investment (time) – A savings account can be opened and closed in a very short time period. Real estate requires a longer time commitment of funds.

6. **Management** (by the owner or a property manager) – A savings account requires little management decision making by the owner. A real estate investment requires substantial investment by the owner or manager. Examples of these decisions are: purchase, sell, make repairs, capital improvements, refinance, etc.

7. **Appreciation** of the investment – Savings accounts often lose purchasing power as a result of inflation. Typically, real estate tends to appreciate over time and can be used as a hedge against inflation.

8. **Income tax advantages** – There are no income tax advantages for savings accounts. All interest is taxed directly. Real property may offer an opportunity to reduce, defer, or eliminate income taxes.

9. **Leverage** – is the borrowing of funds in hopes of earning a return greater than the cost of the borrowed funds. Investors cannot borrow money at a rate which would allow successful investment in a savings account. Investors in real estate can often borrow money at a lower rate than the yield on the real estate investment.

   Leverage can be positive, negative, or neutral. Positive leverage is achieved when funds invested in a property, which has a higher rate of return than the cost of the borrowed funds. Negative leverage occurs when the cost of the borrowed funds are greater than the return of the property investment. Neutral leverage occurs when the cost of the borrowed funds is equal to the return on the property investment.

Real Estate Financing

A. Types of financing for real estate purchases

1. **Cash** – The purchase of real estate may be financed entirely by the purchaser without funds provided by another party. A significant amount of commercial real estate is financed entirely by cash.

2. **Trust deed** (deed of trust) – A legal instrument similar to a mortgage that transfers title of property to a trustee, (sometimes referred to as a deed of trust). The borrower conveys title to a trustee for the benefit of the lender, but retains the right to use and occupy the property. Trust deeds are used to eliminate the need for a foreclosure proceeding against the borrow in the event of default.

3. **Contract for deed** (land contract) or (installment sales contract) – This agreement is also known as a land contract or an installment sales contract. The purchaser agrees to pay a small down payment when the contract is signed,

with the balance in specified amounts over the term of the contract.

4. **Mortgages** (most common type of financing for real estate) - A loan to finance real estate transfers. Many types of mortgages have developed due to the great variety of circumstances relating to the sale and development of real estate (the term mortgage as used here includes trust deeds).

   a. **Conventional** – A loan that is neither insured nor guaranteed by the federal government.

   b. **Insured** – A loan where the lender is insured against loss by an agency of the federal government or a private insurer.

   c. **Construction** – A short-term loan made to finance new construction. Funds are advanced as construction progresses, with permanent financing arranged upon completion.

   d. **Purchase-money-mortgage** – A mortgage given by the buyer to the seller to finance a portion of the sales price of real estate. It is used in place of, or in addition to, institutional financing.

   e. **Junior mortgage** – A mortgage on property executed and recorded after a prior lien has been made.

   f. **Chattel** – A mortgage only on personal property.

   g. **Blanket** – A mortgage covering more than one property. It is a common occurrence with subdivision development property, where one parcel is not sufficient to satisfy the lender's requirements.

h. **Open-end** – A mortgage which allows the borrower to obtain additional funds as long as and agreed loan-to-value ratio is maintained or other specified terms are met.

i. **Wraparound** – A mortgage that is junior to, but includes any existing mortgage on the property. Usually a third party lender refinances the property. This includes assuming the existing mortgage and wrapping around the new subordinate mortgage.

5. Sources of mortgage loans

a. Commercial banks

b. Mutual savings banks

c. Pension funds

d. Profit sharing funds

e. Credit unions

f. Mortgage bankers

g. Insurance companies

h. Individuals

6. Mortgages are classified according to their repayment structure

a. **Fixed rate mortgage** – A permanent loan in which the sum of the principal and interest payments remains fixed throughout the term of the loan. It usually carries a higher interest rate than an adjustable rate loan because there is more risk to the lender.

b. **Variable or adjustable rate mortgage** – A mortgage loan in which the interest rate is periodically adjusted in accordance to some formula or index. It typically carries a lower interest rate initially because there is less risk to the lender compared to a fixed rate loan.

c. **Graduated payment mortgage** – A mortgage loan which requires lower initial payments which will increase after a specified period of time.

d. **Partially amortized mortgage** (uses a balloon payment at the end of the term) – A mortgage loan which is not fully amortized at maturity and requires the total principal to be paid at the end of the mortgage term in one lump sum. The lump sum payment made at the end of the mortgage term is referred to as a balloon payment.

7. The effects of financing on mortgage payments

a. Amount borrowed

b. Interest rate

c. Term of loan

d. Frequency of payments

Primary Rates

A. **The overall yield rate (Y_o), also known as the discount rate (r)**, reflects the return on investment; it consists of four factors in the summation concept.

1. **Safe rate** – The base rate on the safest investments such as government insured investments.

2. **Risk rate** – An amount in addition to the safe rate which compensates the investor for the degree of risk in the investment.

3. **Rate for Non-Liquidity** – An amount in addition to the safe rate and risk rate which compensates the investor for the time necessary to convert the real estate into cash.

4. **Rate for Management** (Management of investment rate) – An amount in addition to the safe rate, risk rate, and non-liquidity rate which compensates the investor for decision making process to management the real estate investment.

B. **Recapture rate** – Provides for the return of the investment in the wasting portion of the asset. This is similar to the depreciation rate for the improvement.

C. **Effective tax rate** – Reflects the relationship between the real estate taxes and the value of the property.

Generic Capitalization Formulas

A. The IRV formula (equation)

1. I (income) = Rate x Value

2. R (rate) = Income ÷ Value

3. V (value) = Income ÷ Rate

B. The VIM (VIF) formula – the VIM (VIF) factor is also known as the gross income multiplier.

1. V(value) = Income x Factor

2. I (Income) = Value ÷ Multiplier

3. M (Multiplier) = Value ÷ Income

Example 1-1

Amount Borrowed Varies

	Lower Loan Ratio	Higher Loan Ratio
Amount Borrowed:	100,000	150,000
Interest Rate:	8.00%	8.00%
Term of Loan (years):	25	25
Partial Payment Factor:	0.007718	0.007718
Monthly Payment: (1)	772	1,158
Total Payment (2)	231,545	347,317
Total Interest Paid (3)	131,545	197,317

1) When the amount borrowed is increased, the monthly payment increases.

2) When the amount borrowed is increased, the total payment increases.

3) When the amount borrowed is increased, the total interest paid increases.

Example 1-2

Interest Rate Varies

	Lower Interest Rate	Higher Interest Rate
Amount Borrowed:	100,000	100,000
Interest Rate:	6.00%	10.00%
Term of Loan (years):	25	25
Partial Payment Factor:	0.006443	0.009087
Monthly Payment: (1)	644	909
Total Payment (2)	193,290	272,610
Total Interest Paid (3)	93,290	172,610

1) When the interest rate increases, the monthly payment increases.

2) When the interest rate increases, the total payment increases.

3) When the interest rate increases, the total interest paid increases.

Example 1-3

Term of Loan Varies

	Shorter Term	Longer Term
Amount Borrowed:	100,000	100,000
Interest Rate:	8.00%	8.00%
Term of Loan (years):	15	25
Partial Payment Factor:	0.009557	0.007718
Monthly Payment: (1)	956	772
Total Payment (2)	172,017	231,545
Total Interest Paid (3)	72,017	131,545

1) When the term of the loan increases, the monthly payment decreases.

2) When the term of the loan increases, the total payment increases.

3) When the term of the loan increases, the total interest paid increases.

Example 1-4

Frequency of Payments Varies

	Monthly Payments	Annual Payments
Amount Borrowed:	100,000	100,000
Interest Rate:	10.00%	10.00%
Term of Loan (years):	20	20
Partial Payment Factor:	0.00965	0.11746
Monthly Payment: (1)	965	11,746
Total Payment (2)	231,605	234,919
Total Interest Paid (3)	131,605	134,919

1) When the frequency of payments is increased, the payment decreases.

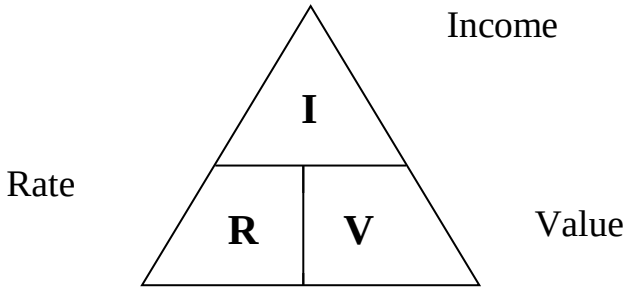
2) When the frequency of payments is increased, the total payment decreases.

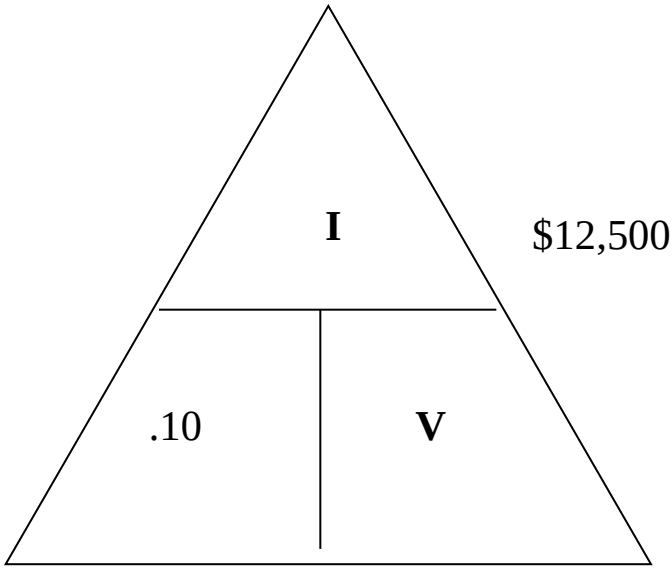
3) When the frequency of payments is increased, the total interest paid decreases.

Example 1-5

IRV

A corner lot in a business district is used as a parking lot. The monthly income earned by the property is $12,500. The owner's required rate of return for this type of property is 10% per year. What is the value of the property?





I = $150,000
$12,500 x 12 = $150,000

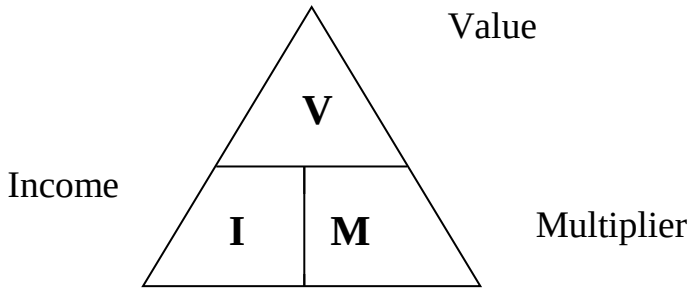
$150,000 divided by .10 = $1,500,000

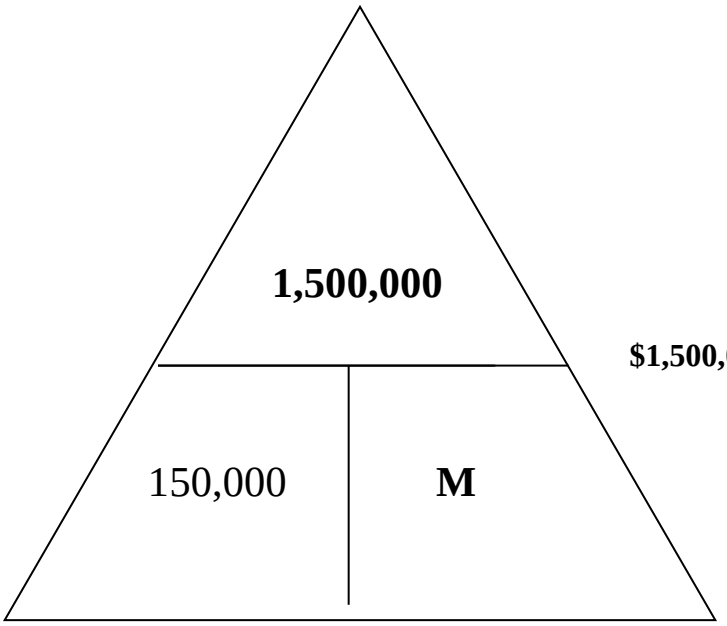
Example 1-6

VIM (VIF)

You derived the value of $1,500,000 for the corner parking lot in a business district in Example 1-5. You calculated the annual income earned of $150,000. Using VIM, calculate the income factor for this property.

Note: The VIM (VIF) factor is known as the Gross Income Multiplier (GIM).





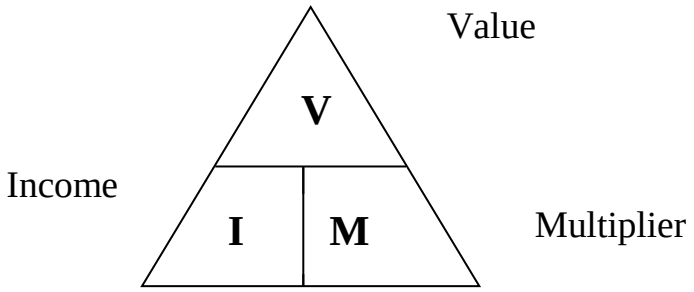
$1,500,000 divided by $150,000 = 10 (Factor)

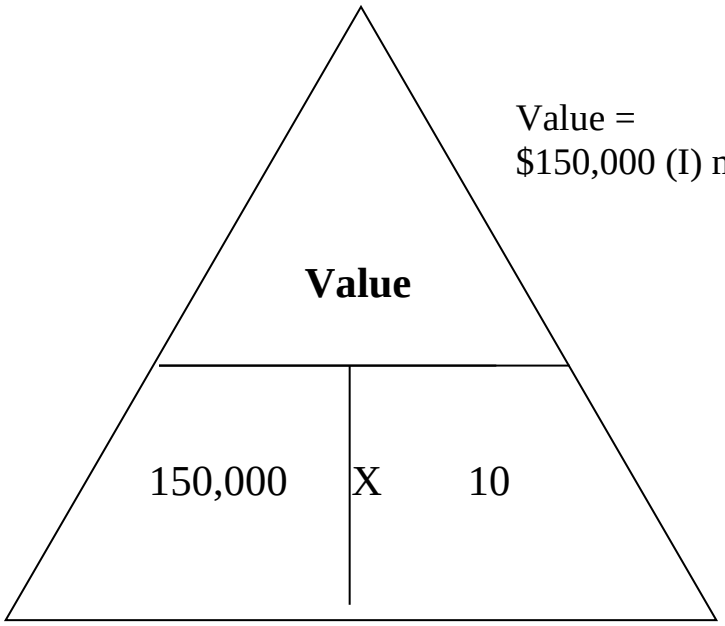
Example 1-7

VIM (VIF)

You derived a factor of 10 that is applicable to the corner parking lot in the business district in Example 1-6. You calculated the annual gross income earned of $150,000. Using VIM, calculate the value for this property.

Note: The VIM (VIF) factor is known as the Gross Income Multiplier (GIM).





Value =
$150,000 (I) multiplied by 10 (F) = $1,500,000

Review Questions

1. One of three approaches to value in which the appraiser derives a value indication by converting anticipated benefits through ownership of an income-producing property is the _____ approach.

2. Real estate competes with other investments for the investor's dollar. As an investor analyzes various opportunities what will she consider?

   1._____

   2._____

   3._____

   4._____

   5._____

3. The economic principle of _____ states that value is created by the expectation of benefits to be derived in the future.

4. The economic principle of _____ states that a property's maximum value tends to be set by the lowest cost or price at which another property of equivalent utility can be acquired.

5. Competition is created by the potential for profits. However, competition among sellers may lead to a/an _____, which reduces prices and profits. Competition among buyers may lead to a/an _____, which increases prices and profits to sellers.

6. The _____ rate reflects the return of the investment in the wasting asset.

7. List nine factors influencing the behavior of investors.

1 _____ 6 _____

2 _____ 7 _____

3 _____ 8 _____

4 _____ 9 _____

5 _____

8. _____ leverage is achieved when funds are invested in property, which has a higher rate of return than the cost of the borrowed funds.

9. The four most common methods to finance real estate are:

1_____

2_____

3_____

4_____

10. A mortgage on personal property is called a _____ .

11. A _____ is junior to, but includes any existing mortgage on the property.

12. List the four types of mortgages classified according to repayment provisions.

    1 _____ 3 _____

    2 _____ 4 _____

13. A payment on the balance due of a note at the end of the loan term that is in excess of the regular payment amounts is called a _____ .

14. A loan that is neither insured nor guaranteed by the federal government is called a _____ mortgage.

15. The four variables in real estate financing that affect the mortgage payment are:

    1 _____ 3 _____

    2 _____ 4 _____

16. The _____ rate reflects the return on the investment.

17. The summation concept is a theoretical procedure in developing the discount rate for a real estate investment and is comprised of the following four parts.

    1 _____ 3 _____

    2 _____ 4 _____

18. The _____ reflects the relationship between real estate taxes and the value of the property.

19. To obtain value using the IRV formula, one must _____ the income by the rate.

20. To obtain value using the VIM (VIF) formula, one must _____ the income by the multiplier.

Chapter 2

Objectives

Upon completion of this chapter, you will be able to:

- ✓ Identify the sources of data the appraiser utilizes in estimating income and expenses and explain the importance of the accuracy and relevance of the market information.
- ✓ Identify and explain the various types of rents associated with income-producing properties.
- ✓ Identify and understand leases associated with the rental of income-producing properties.
- ✓ Understand and be able to derive rental units of comparison in the application of the income approach.
- ✓ Explain the levels of income, potential gross income, effective gross income, and annual net operating income; and the importance of accurate income and expense analysis.
- ✓ Identify the categories of income and expense used in reconstructed operating statements of income-producing properties.
- ✓ Develop an accurate reconstructed operating statement for appraisal purposes.

Introduction

This chapter is designed to provide students with an understanding and working knowledge of the sources used by appraisers to obtain income and expense data. The chapter also includes an introduction to leases and rents used to help estimate income. The student is exposed to the levels of income (potential gross income, effective gross income, and net operating income), along with proper and improper expenses to consider when estimating the appropriate net operating income to capitalize in the income approach.

Sources of Income Data

A. Market sources of data

1. Sales (deeds) - which usually have a stated price consideration or tax stamps that indicate amount of sale.

2. Leases - contracts that spell out all rights, privileges and obligations of landlords and tenants.

3. Offers, listings, opinions, and other market transactions, can be a valid source of market information, if care is exercised in their use.

4. Sources of rental income data:

   a. Commercial property managers
   b. Tenants of comparable properties
   c. Real estate brokers and sales people
   d. Sellers and purchasers of comparable properties
   e. Public records

B. Established data banks

1. Organizations - credit bureaus, real estate boards, Multiple Listing Services, tax map reproduction firms, and International Association of Assessing Officers data bank.

2. In-house data banks

   a) Income data factor file

      1. Result of income data bank.

      2. Storing of collected data in an accessible format where it may be retrieved by property type and class and used in the mass appraisal process.

(a) Sales price
(b) Overall rates
(c) Rental information
(d) Expense data
(e) Interest rates
(f) Vacancy rates

C. Published sources of data

1. *BOMA Experience Exchange Report: An Income/Expense Analysis for Office Buildings,* published by Building Owners and Managers Association.

2. *Income/Expense Analysis, Apartments, Condominiums and Cooperatives,* published by the Institute of Real Estate Management.

3. *Dollars and Cents of Shopping Centers* published by the Urban Land Institute; provides operating results for all types of shopping centers.

4. *Trends in the Hotel Industry,* published by Pannell Foster.

5. *Hospitality Market Data Exchange* (hotel/motel); published by Hospitality Market Data Exchange of Mineola, NY

D. World Wide Web (Internet)

Basis of Income

A. Rental information may be recorded on leases that are a matter of public record. Leases ordinarily detail the terms and other important considerations between the tenant and the property owner.

B. Listed below are various types of leases. In the marketplace, a lease may be a combination of two or more of these various types of leases.

1. Month-to-Month Lease - Short term leases that may or may not be in written form. This type of lease provides no security for the tenant or the landlord.

2. Short-Term Lease - Generally written with the terms and provisions of the lease detailed. This type of lease is generally for a period of less than 10 years, and is usually for a period of less than 5 years.

3. Long-Term Lease - Provides for terms extending more than 10 years.

4. Graduated (step-up or step-down) - Provides for changes as called for in the lease agreement at one or more points during the lease term. Also known as a step-up or step-down lease.

5. Percentage Lease - A contract which calls for a fixed minimum base rent and a variable rent which is based on volume of business, sales, productivity, or use of the property by the tenant.

6. Gross Lease - The landlord is required to pay all operating expenses associated with the real estate. To the landlord, the income from a gross lease is considered gross income.

7. Net Lease - The tenant is required to pay all operating expenses associated with the real estate. To the landlord, the income from a net lease is considered net income.

C. Types of Rents

1. Market rent - The amount of rent that a property should command in the open market. Market rent should reflect the location of the property, size of the property, supply and demand factors, and terms of lease between knowledgeable and prudent owners and tenants. (Also known as economic rent).

2. Contract rent - The actual rent paid, as required in the contract or lease.

3. Excess rent - The amount which contract rent exceeds market rent. This could be caused by a locational

advantage, unusual management, uninformed tenants, or other economic factors.

4. Percentage rent - The amount of rent received in accordance with the terms of a percentage clause in a percentage lease contract.

   a) Minimum base rent - The fixed portion of the rent under the terms of a percentage lease.

   b) Overage rent - The variable portion of the rent under the terms of a percentage lease. This is the rent over and above the *minimum base rent.* (Typically, the overage rent is based on a percentage of sales or revenue from the business conducted on the premises.)

5. Leasehold rent (leasehold income) - Occurs when market rent is greater than contract rent. In effect, the tenant is receiving as income the amount of the difference between contract rent and market rent.

Considerations When Comparing Rental Properties

A. Date of lease - For purposes of determining market rent, it is important when comparing rental properties, that the date of the lease agreement be recent and representative of current market conditions.

B. Location of property - It is important that the properties used as rent comparables, for the subject, be located in similar locations or subject to the same economic influences.

C. Physical characteristics of property - Properties used as rent comparables for the subject should be similar in age, size, condition, quality, desirability, and other amenities, etc.

D. Terms of lease - It is important that the properties used as rent comparables for the subject be influenced by the same or similar terms, such as who pays maintenance expenses, taxes, insurance, or other special agreements. Examples of special agreements are: expense stops, renewal options, build-out allowances, or other inducements to rent.

Rental Units of Comparison

A. Rent per apartment unit

B. Rent per room

C. Rent per space

D. Rent per square foot

1. Gross leasable area (GLA) - includes the entire area of the building.
2. Net leasable area (NLA) - includes only the floor area occupied by the tenant.

E. Gross Income Multiplier (GIM) defined - the relationship between gross income and the value of the property, which is generally represented by its sale price. The income is the annual gross income.

a) A GIM is generally used with commercial and apartment properties.

b) The GIM is calculated by dividing the sale price (value) by the annual gross income. For example, assume a sale price of $2,400,000 for an apartment that has an annual gross income of $279,000. $2,400,000 / 279,000 = 8.60 (GIM).

Income and Expense Statement

A. Reconstructed from owner's statement - Often the appraiser will use the information from the owner's operating statement and reconstruct an income statement for the property, based on the current market rents and typical proper expenses for one year.

B. Developed from rent comparables in the area.

C. Format of income and expense statement.

1. Gross income estimate (potential gross income - PGI).

a) Potential gross income is the annual economic rent for the property at100 % occupancy.

b) It includes the market rent from all scheduled sources.

2. Vacancy and collection loss (derived from market)

a) Typically, all properties will experience some amount of vacancy during their economic life.

b) During the economic life of a property, there will be a loss of rental income because of the failure of tenants to pay the rent.

c) The amount that is allowed for vacancy and collection loss must be based on an analysis of properties that are similar to the subject.

3. Miscellaneous income

1. Miscellaneous income may come from several sources other than the scheduled rents.

2. Miscellaneous income covers all income generated by the operation of the real property that is not derived directly from scheduled rental of space.

3. Some examples of miscellaneous income are parking fees, vending machines, coin operated laundries, and cable installation. This income may or may not be attributable to the real property. The appraisers must consider this as business income, or as income to the real property, depending on the market.

4. Effective gross income (EGI)

a) Effective gross income is obtained after determining the potential gross income for the property.

b) EGI is PGI less vacancy and collection loss plus appropriate miscellaneous income.

5. Proper expenses

a) Operating expenses are ordinary and typical expenses that are necessary to keep the property functional and rented competitively with other properties in the area.

(1) Operating expenses vary from property to property, depending on the type of occupancy, use type, and quality of management.

(2) In analyzing the operating expenses for a property, the operating statements from comparable properties must be reviewed and the following considered:

(a) Does the expense amount appear to be typical for the property in question, and is the amount substantiated by expense statements of comparable properties?

(b) Do the expenses tend to appear infrequently?

(c) Do the expenses appear to indicate typical management?

(d) Do the expenses indicate a specific weakness of the property in question?

(e) Are the various reported expenses consistent as they relate to each other (maintenance, age, and reserves for replacement)?

(f) Is the ratio of expenses to effective gross income comparable to those for competitive properties?

(3) Examples of proper expenses are:

(a) Management

(b) Insurance

(c) Utilities

(d) Maintenance

(e) Legal and accounting fees

(f) Employee wages and fringe benefits

(g) Yard care

(h) Miscellaneous

(i) Reserves for replacement

b) Reserves for replacement

(1) Short-lived items are items which are expected to have a remaining economic life less than the remaining economic life of the property.

(2) The amount of expense for short-lived items is computed by dividing the replacement cost of the item by the number of years of total economic life.

(3) Some examples are carpets, drapes, heating, air conditioning, ranges, refrigerators, water heaters, and paving, etc.

6. Improper expenses

a) Depreciation

(1) Depreciation will be considered in the income approach as recapture and handled as part of the capitalization rate.

(2) Depreciation found on operating statements for income tax purposes is an accounting provision for the recovery of cost of an asset. It is generally not the same as recapture provision in the capitalization rate.

b) Debt service
(principle and interest on mortgage)

(1) Principal and interest expense is a financing expense which is a part of the cost of capital necessary to acquire the property.

(2) Normally, a component for the cost of capital (the discount rate) is included in the capitalization rate.

c) Owner's personal expenses (income taxes, etc.)

(1) Any business expenses not necessary for generation of income to the property are not valid operating expenses of the property.

(2) The income tax shown on the owner's personal income, not the value of the property.

d) Capital improvements

(1) Capital improvements are not items of annual expense. Typically, capital improvements will result in either

increases in property value, economic life of the property, or income to the property.

(2) For example, the addition of a swimming pool to an apartment complex would not be considered a normal annual operating expense of the property.

e) Franchise fees and special corporation costs

(1) A franchise is usually an exclusive right to furnish public services or to sell a particular product in a certain community.

(2) Normally franchise fees, special corporation costs, and items of this nature are charges against the owner of the property or the business, rather than the property itself, thus would not normally be considered operating expenses of the real estate.

7. Real Estate Taxes - are a valid operating expense of property. However, because the ad valorem tax appraiser is appraising the property for the ultimate purpose of determining the amount of ad valorem taxes, the appraiser must not make a deduction for real estate taxes. It is not possible to know the proper amount of real estate taxes until the amount of the assessed value is known. Thus, to make a deduction for real estate taxes, the appraiser would have to pre-suppose the amount of value in order to compute the amount of real estate taxes. The ad valorem tax appraiser avoids this problem by making no deduction for the real estate taxes from the income. Instead the appraiser includes an effective tax rate as a component of the capitalization rate for the property. (Concept is similar to depreciation as previously discussed)

a) Proper as an expense under certain conditions, such as appraising property for purposes other than ad valorem taxation.

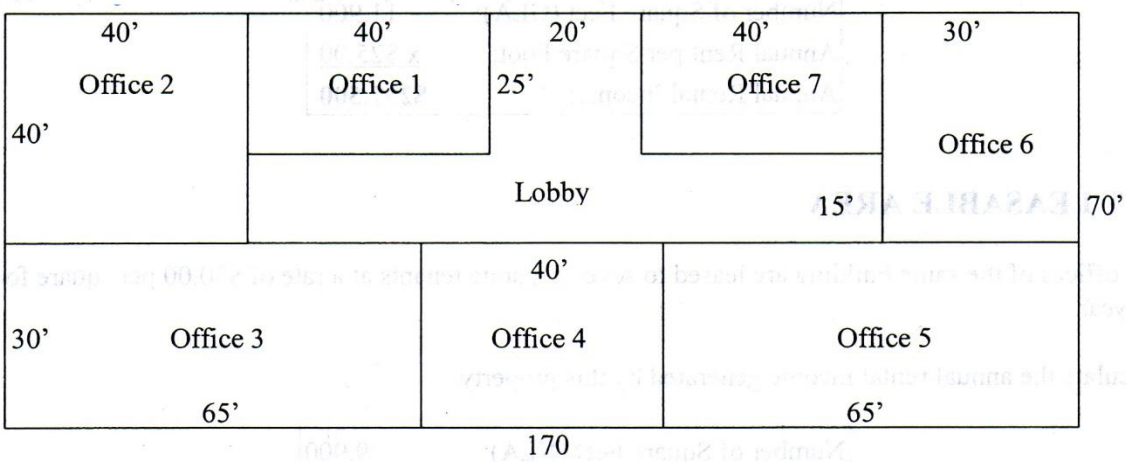
b) Not proper as an expense under certain conditions, such as ad valorem taxation.

Example 2-1

Gross and Net Leasable Area

When using the square foot unit of comparison on office buildings and shopping centers, care must be exercised in the comparison process. Some leases refer to gross leasable area, but other leases are negotiated on the basis of net leasable area. The gross leasable area (GLA) includes all common areas, such as halls, restrooms, and vestibules. The net leasable area (NLA) includes only the floor area actually occupied by the tenant.

The floor plan below of a one-story office building will be used to demonstrate the proper procedures in developing GLA and NLA units of comparison.



Gross leasable area = 70' x 170' = 11,900 GLA square feet

Net leasable area is the building area only using floor space occupied by the tenant

Office 1	40 x 25 =	1,000	
Office 2	40 x 40 =	1,600	
Office 3	30x65 =	1,950	
Office 4	40 x 30 =	1,200	
Office 5	30x65 =	1,950	
Office 6	30 x 40 =	1,200	
Office 7	40 x 25 =	1,000	
	NLA =	9,900	square feet

Example 2-1 (continued)

Gross and Net Leasable Area

Gross Leasable Area

The building contains 11,900 square feet and is leased on a basis of $25.00 per square foot of gross leasable area per year.

Calculate the annual rental income generated by this property.

Number of Square Feet (GLA):	11,900
Annual Rent per Square Foot:	x $25.00
Annual Rental Income:	**$297,500**

Net Leasable Area

The offices of the same building are leased to seven separate tenants at a rate of $30.00 per square foot per year.

Calculate the annual rental income generated by this property.

Number of Square Feet (NLA)	9,900
Annual Rent per Square Foot:	x $30.00
Annual Rental Income:	**$297,000**

In using the NLA rent per square foot, only the area leased by the tenant can be used to develop the annual rent. If the NLA rent of $30.00 per square foot is multiplied by the gross area of the building, the indicated potential gross income would be $357,000 and would result in a substantial error in the value estimate utilized in the capitalization process.

Only one type of rent may be used when developing rent per square foot as a unit of comparison. It is necessary that all rents be expressed in terms of either gross leasable area or net leasable area.

Exercise 2-1

Units of Comparison – Rent per Square Foot

PROBLEM:

You are developing a rent per square foot as a unit of comparison for small office buildings from an analysis of the market. You have obtained the following information:

A comparable sale has been identified that is similar to the subject in terms of age, size, construction, condition, and location. It is a two-story office building and has a gross leasable area (GLA) of 16,000 square feet. The common area contains 3,000 square feet and includes all hallways, elevators, stairs, and includes the reception area on the ground floor. The building rents for $380,000 per year.

Analyze the rent schedule and develop rent per square foot as a unit of comparison, on both a gross square footage and net square footage basis.

Exercise 2-2

Units of Comparison – Apartment Unit vs. S.F.

Five rental properties have been located in the neighborhood in which you are appraising apartment complexes. All five properties are comparable in age, condition and location.

Analyze the rental data to determine the range of values on a per unit and per square foot basis for each type of unit. Discuss the merits of both approaches. Which method is the better choice for determining the monthly potential gross income of comparable buildings in this neighborhood?

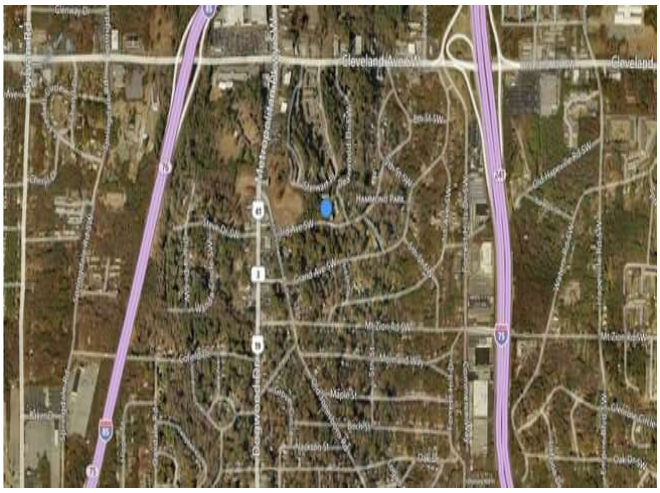
	One Bedroom Unit		Two Bedroom Unit Size	
	Size	Monthly Rent	Size	Monthly Rent
Rental 1	575	$ 820	810	$890
Rental 2	550	740	840	940
Rental 3	580	840	850	1,000
Rental 4	535	750	775	870
Rental 5	525	720	780	860

Exercise 2-3

Development of Market Rental Estimate

You have been asked to appraise The Villas at Henry Crossing. This 500 unit property is 490,000 total square feet, five years old and has two swimming pools and two clubhouses. Covered parking is provided for every unit. The breakdown of the complex is listed in the table below. Both the aerial photograph and front photo of the property have been provided to assist you in valuing the subject.

You have researched the area and found five comparable apartment complexes similar to the subject. Analyze the rental information on the following page and develop both the range of rents and an estimate of market rent for the subject property.



Villas at Henry Crossing					
Floor Plan	Bed	Bath	Sq, Ft. Per Unit	Number of Units	Total Sq. Ft.
Plan A	1	1	800	300	240,000
PlanB	2	1.5	1,250	200	250,000
Total				500	490,000

Exercise 2-3

Development of Market Rental Estimate (Continued)

Alexan Palm Valley						Unit	Rent
Floor Plan	Bed	Bath	Sq. Ft.	Rent	Parking	Count	Per Sq. Ft.
Sago Plan	1	1	760	840	Covered	150	1.11
The Queen Plan	2	2	1,125	1,080	Covered	150	0.96
Total			282,750			300	
Amenities: One year old and has one pool and one clubhouse, covered parking is included							

Wolfe Creek Lane						Unit	Rent
Floor Plan	Bed	Bath	Sq. Ft.	Rent	Parking	Count	Per Sq. Ft.
The Diamond	1	1	760	850	Covered	150	1.12
The Challenger	2	2	1,250	1,320	Covered	90	1.06
Total			226,500			240	
Amenities: Brand new and has one pool and one clubhouse, covered parking is included							

Park Hill Apartments						Unit	Rent
Floor Plan	Bed	Bath	Sq. Ft.	Rent	Parking	Count	Per Sq. Ft.
Plan 1	1	1	850	925	Covered	300	1.09
Plan 2	2	2	1,250	1,200	Covered	200	0.96
Total			505,000			500	
Amenities: Five years old and has two pools and two clubhouses, covered parking included							

Rock Pointe						Unit	Rent
Floor Plan	Bed	Bath	Sq. Ft.	Rent	Parking	Count	Per Sq. Ft.
Plan A	1	1	850	895	Covered	200	1.05
Plan B	2	2	1,025	985	Covered	100	0.96
Total			272,500			300	
Amenities: Five years old and has two pools and one clubhouse, covered parking is included							

Stone Creek at Arrowhead						Unit	Rent
Floor Plan	Bed	Bath	Sq. Ft.	Rent	Parking	Count	Per Sq. Ft.
Plan 1	1	1	700	770	Covered	210	1.10
Plan 2	2	1	1,050	990	Covered	100	0.94
Total			252,000			310	
Amenities: Five years old and has two pools and one clubhouse, covered parking is included							

Example 2-2

Outline of Income & Expense Statement

	POTENTIAL GROSS INCOME
-	Vacancy & Collection Loss
+	Miscellaneous Income
=	**EFFECTIVE GROSS INCOME**
-	Allowable Expenses (Operating Expenses & Reserves for Replacement)
=	**NET OPERATING INCOME**

Outline of Income & Expense Statement

Exercise 2-4

Developing a Potential Gross Income Estimate

You are appraising an apartment building with three levels. The first level contains garden level apartments with 4 one-bedroom units, 2 two-bedroom units, and 2 three-bedroom units. The second level contains 6 one-bedroom units and 4 two-bedroom units. The third level contains 4 two-bedroom units and 4 three-bedroom units.

Market analysis provides:

First level one-bedroom units – Market rent $700.00 per month

First level two-bedroom units – Market rent $800.00 per month

First level three-bedroom units – Market rent $900.00 per month

Units located on the second and third levels rent for 25% more than those on the first level. The janitor lives in a one-bedroom unit on the first level, which is rent free and part of the salary. The owners of the building occupy a three-bedroom unit on the third level and pay no rent.

Develop a Potential Gross Income (PGI) estimate for this apartment building.

Example 2-3

Vacancy & Collection Loss

It is not likely a property will remain fully rented, or that all of the rents will be collected, during its economic life. A vacancy and collection loss factor is determined so that it may be applied to the potential gross income. The vacancy and collection factor is determined by a study of other properties that are similar to the subject being appraised.

The following is an example of developing vacancy rates for apartments.

Complex	Units	Occupied	Vacant	Vacant / Total Units	Vacancy Rate
The Villas	142	132	10	10 / 142	0.0704
Crabapple	144	134	10	10 / 144	0.0694
Pine Cove	128	119	9	9 / 128	0.0703

The indicated vacancy rate is **0.0700 or 7%**.

The following is an example of developing a rate for collection loss.

Complex	Rents Receivable	Collected	Collection Loss	Loss / Billable	Rate
The Villas	153,000	150,000	3,000	3,000 / 153,000	0.0196
Crabapple	160,000	156,800	3,200	3,200 / 160,000	0.0200
Pine Cove	132,000	129,300	2,700	2,700 / 132,000	0.0205

The indicated rate for collection loss is **.0200 or 2%**.

The indicated rate for vacancy and collection loss is **9%**.

Exercise 2-5

Income & Expense Statement – Vacancy Rate

You are reconstructing an income and expense statement for an apartment complex containing 72 units. You have found four apartment complexes having similar characteristics and amenities, located in the same neighborhood as that of your subject.

Complex	Units	Occupied
Briargate	80	76
Creekside	60	57
Beachfront	78	74
Lakside	62	59

What vacancy rate would be appropriate for your reconstructed income and expense statement?

Exercise 2-6

Income & Expense Statement – Collection Loss Rate

You are reconstructing an income and expense statement for an apartment complex containing 120 units, and need to determine a rate for collection loss. You have found four apartment complexes with rental information, having similar characteristics and amenities, located in the same neighborhood as your subject.

Complex	Number of Units	Rents Receivable	Rents Collected
Peachtree	108	984,900	965,100
Applewood	120	1,094,400	1,072,600
Elderberry	110	1,056,000	1,035,200
Brentwood	124	1,130,800	1,107,900

What collection loss rate will you use in your reconstructed income and expense statement?

Exercise 2-7

Income & Expense Statement – Vacancy and Collection Loss Rate

You are reconstructing an income and expense statement for a 180 unit apartment complex located just south of the central business district. The complex is eight years old and has typical amenities and management.

The following five apartment complexes are similar to the subject in management, age, condition, etc.

Complex A:

Contains 210 units with 199 occupied. The billable rent was $1,925,300 with the rent collected being $1,886,800.

Complex B:

Contains 180 units with 171 occupied. The billable rent was $1,744,200 with the rent collected being $1,709,500.

Complex C:

Contains 196 units with 186 occupied. The billable rent was $1,899,200 with the rent collected being $1,861,200.

Complex D:

Contains 200 units with 190 occupied. The billable rent was $1,938,000 with the rent collected being $1,898,000.

Complex E:

Contains 220 units with 209 occupied. The billable rent was $2,131,800 with the rent collected being $2,089,300.

Use the above data to determine the rate you will use for vacancy and collection loss in your reconstructed income and expense statement.

Example 2-4

Reconstruction of the Income and Expense Statement

You have been given the owner's operating statement for a 60-unit apartment complex. The statement has been prepared by the owner's accountant for income tax purposes and you will need to reconstruct it for appraisal purposes.

Income and Expense Statement Peachtree Apartments

Income		
Apartment rents		$581,400
Laundry facilities		7,500
	Gross income	$588,900
Expenses:		
	Real estate taxes	$45,450
	Insurance	30,600
	Salaries	34,500
	Fringe benefits	9,650
	Painting 10 units	20,000
	Utilities	73,100
	Grounds maintenance	18,500
	Advertising	4,800
	Depreciation	195,000
	Debt service (principal & interest)	198,400
	Replace 5 refrigerators	4,000
	Replace 8 stoves	5,600
	Replace 10 water heaters	6,000
Total expenses:		**$645,600**

Analysis of the market indicates a potential gross income (PGI) of $630,000, and a rate of 5% for vacancy and collection loss. The owner's miscellaneous income is typical and the allowable expenses are similar to comparable complexes, thus requiring no adjustment. Also, expenses for items that are considered reserves for replacement are comparable to similar complexes. The owner did not include any allowance for management. However, a management fee of 5% of the effective gross income is an appropriate adjustment.

The stoves and refrigerators have a 15-year life, while the water heaters have a 10-year life. There is one stove, refrigerator and water heater in each apartment. The roof cover for the complex has a 20-year life and would cost $60,000 to replace. All units should be repainted on a five-year cycle. The floor covering in each unit lasts 9 years and costs $1,200 per unit to replace.

Using market data and the owner's allowable expenses you can now reconstruct the income and expense statement.

Potential Gross Income			**630,000**	100.00%
Vacancy and Collection Loss			**(31,500)**	5.00%
Miscellaneous Income			**7,500**	
Effective Gross Income			**606,000**	
Operating Expenses				
Management	30,300	5.00%		
Insurance	30,600	5.05%		
Salaries	34,500	5.69%		
Fringe Benefits	9,650	1.59%		
Utilities	73,100	12.06%		
Ground maintenance	18,500	3.05%		
Advertising	4,800	0.79%		
Reserves for Replacement				
Refrigerators [(800 x 60) / 15]	3,200	0.53%		
Stoves [(700 x 60) / 15]	2,800	0.46%		
Water heaters [(600 x 60) / 10]	3,600	0.59%		
Painting [(2,000 x 60) / 5]	24,000	3.96%		
Floor cover [(1,200 x 60) / 9]	8,000	1.32%		
Roof cover (60,000 / 20)	3,000	0.50%		
Total Expenses			**(246,050)**	40.60%
Net Operating Income			**359,950**	59.40%

To assist the appraiser in comparing expenses with those of other properties and assist in developing benchmarks for mass appraisal of income-producing properties, expense ratios have been provided.

Analysis of the Reconstructed Income and Expense Statement

Potential Gross Income: This is market rent or economic rent as determined from an analysis of similar properties. The owner's statement reflected the rents collected not potential gross income (PGI). PGI is market rent at 100% occupancy. If there is a difference between market rent and contract rent, market rent must be used to develop PGI.

Vacancy and Collection Loss: It is highly unlikely a property will remain fully rented, or that all of the rents will be collected, during its economic life. Thus, a vacancy factor and/or a factor for collection loss must be determined by a study of other properties, similar to the subject being appraised. The owner reported his rents collected, but did not provide any information on vacancies or collection loss amounts. Since the purpose of reconstructing the income and expense statement is to reflect typical income and expenses compared to similar properties, an adjustment for vacancy and collection loss is appropriate. Market analysis indicated an adjustment of 5% was typical, so this was the amount that was applied in reconstructing the owner's statement.

Miscellaneous Income: The owner did have miscellaneous income and listed it separately on the statement as income from his coin-operated laundry facilities. Similar units with coin-operated laundry facilities reported similar incomes, thus the owner's listed amount was considered typical.

When considering miscellaneous income and analyzing owner's operating statements for various use-types of properties, the appraiser must make a determination as to whether the miscellaneous income is attributable to real estate or if it might be a part of the business income.

Proper Expenses:

> **Management**: Management fees are a proper expense for this type of property even though the owner did not list an expense for management. A review of similar apartment complexes indicated a fee of 5% of effective gross income was appropriate for apartment complexes in this area.

Insurance: Insurance is a proper expense. However, when analyzing insurance expenses, the premium should reflect the cost for one year. If the premium is for something more or less than one year, it must be adjusted to reflect an annual premium.

Salaries and fringe benefits: Salaries and fringe benefits necessary to maintain the rental income stream to the property are a proper expense. The owner's expense for salaries and fringe benefits was considered typical and the entire amount was allowed.

Utilities: Utilities paid by the landlord are considered a proper expense.

Ground maintenance: Maintaining the grounds, which includes lawn care is a proper expense in the operation of an apartment complex.

Advertising: Advertising is a proper expense. However, a comparison to similar properties should be made to determine if the amount on the owner's statement is too high or too low.

Refrigerators: This item should be considered as a reserve for replacement. The proper allowance can be determined by first multiplying the cost of one refrigerator by the total number of units. Next, divide this amount by the total life for the refrigerators.

Stoves: This item should be considered as a reserve for replacement. The proper allowance is calculated in the same manner as the calculation for refrigerators.

Water Heaters: Water heaters should be considered as a reserve for replacement and is treated the same as refrigerators and stoves.

Painting: Painting should be considered as a reserve for replacement. The proper allowance is determined by multiplying the cost to paint one unit by the total number of units in the complex and then dividing the result by the cycle for painting. In this statement, painting is on a five-year cycle.

Floor Cover: This item was not included in the owner's operating statement. However, market analysis indicates floor coverings last nine years and the cost to replace the floor cover in an individual unit is $1,200. The proper allowance is calculated in the same manner as the calculations for refrigerators, stoves, and water heaters.

Roof Cover: This is another item that did not appear on the owner's statement. However, it should be considered as a reserve for replacement. The proper allowance is calculated by dividing the cost to replace the roof cover by its total life.

Improper Expenses:

These items were listed as expenses on the owner's statement. However, they are not proper expenses and should not be used in developing an estimate of net operating income. Thus, they were omitted from the reconstructed income and expense statement.

Income Taxes: Income taxes are not a legitimate operating expense, because the tax is based on the personal income of the owner.

Depreciation: Depreciation is recognized in the income approach by the recapture rate which is a component of the overall rate or the building capitalization rate.

Debt Service: This includes principal and interest of the mortgage. These items are considered in the capitalization rate in the form of a discount rate.

Real Estate Taxes: Real estate taxes are an allowable expense. However, since we are appraising for ad valorem tax purposes, we do not deduct the taxes as an operating expense, but allow them as a component of the capitalization rate.

Exercise 2-8

Reconstruction of an Income Statement

You have been given an apartment owners operating statement to analyze for a 28-unit apartment complex. The figures are validated by check stubs and you believe them to be accurate. You are required to determine which figures to use as stated, which figures to pro-rate, and which figures to totally eliminate. Mark an "X" in the appropriate column to indicate what should be done with each item of expenses.

	Expense Item	Use as Stated	Pro-Rate	Eliminate
A	Management fee			
B	Repairs			
C	Miscellaneous			
D	Utilities			
E	Interest on mortgage			
F	Principal on mortgage			
G	New roof			
H	Insurance fire (3-year policy)			
I	Insurance liability (1-year policy)			
J	Employee's health policy (1-year policy)			
K	Janitor's salary			
L	Painting exterior			
M	Purchase of 3 new refrigerators			
N	Purchase of 4 range/ovens			
O	Supplies			
P	Corporate income taxes			
Q	Red Cross donation			
R	Carpet replacement (4 units)			
S	Redecorate 7 apartment units			
T	Real estate taxes			

Exercise 2-9

Reconstruction of an Income Statement

Your assignment is to appraise a 30-unit apartment complex. You have the owner's operating statement prepared by the accountant. After careful analysis, you decide that all items are essentially correct. However, the owner did not include certain items that you believe need to be considered. The owner did not include an allowance for vacancy and collection loss, which you believe should be 4%. Also, the owner manages the property and did not include a management expense. However, you believe the typical management expense for this type of property is 5%. Painting and decorating should last 3 years.

Income or Expense Item	Owner's Statement	Reconstructed Statement
Gross Income	365,000	
Vacancy & Collection Loss	-	
Effective Gross Income	365,000	
Expenses:		
Management	-	
Employee's Salaries	34,022	
Employee's Benefits	2,160	
Insurance	7,000	
Gas	13,445	
Painting & Decorating (5units)	6,000	
Repairs	8,425	
Supplies	2,040	
Electricity	4,850	
Water	1,680	
Real Estate Taxes	32,550	
Depreciation	56,000	
Interest on Mortgage	120,650	
Legal & Accounting Fees	4,800	
Principal on Mortgage	19,100	
Miscellaneous Expense	3,150	
Total Expenses	315,872	
Net Operating Income	49,128	

Exercise 2-10

Reconstruction of an Income Statement

You are appraising a 12-year old apartment building and have been given the owner's income and expense statement for last year. This statement shows the actual amounts received and paid out as shown below:

Income or Expense Item	Owner's Statement	Reconstructed Statement
Gross Income	697,500	
Vacancy & Collection Loss	-	
Miscellaneous Income (annually)	27,500	
Effective Gross Income	700,250	
Expenses:		
Management	-	
Interest on Mortgage	168,238	
Insurance	4,500	
Real Estate Taxes	112,500	
Janitor's Salary	36,000	
Water	11,000	
Electricity	16,000	
Natural Gas	18,000	
Miscellaneous Repairs	11,250	
Supplies	3,375	
Principal on Mortgage	70,000	
Exterior Repairs	31,250	
Depreciation	72,000	
Building Addition	120,000	
6 Refrigerators	10,500	
8 Ranges	12,000	
2 Water Heaters	4,500	
Roof Cover		
Floor Cover		
Redecorating		
Total Expenses	701,113	
Net Operating Income	23,887	

Exercise 2-10

Reconstruction of an Income Statement

The apartment complex consists of five buildings with 10 units per building. Each building contains two efficiency units, six 1-bedroom units, and two 2-bedroom units. One water heater serves each building and each unit contains a range and refrigerator.

An analysis of similar apartment complexes provides an indication of monthly market rent as follows:

Efficiency units:	$1,000
1-bedroom units:	$1,250
2-bedroom units:	$1,500
Miscellaneous income:	$27,500 (annual)

Your analysis also indicates a vacancy and collection loss of 5% and a management allowance of 4% of the effective gross income for the subject property. Proper operating expenses are typical, needing no adjustment. Reserves for replacement include water heaters, ranges and refrigerators, roof cover, floor cover, and redecorating. The costs shown on the owner's statement for ranges, refrigerators, and water heaters are representative of the market.

Further analysis indicates a 10-year life for ranges and refrigerators, and a 5-year life for water heaters. Replacement of the roof cover for the entire complex costs $45,000 and has a 12-year economic life. Replacement of floor covers for the entire complex costs $37,500 and has an economic life of five years. Each apartment unit requires redecorating, on average, every five years. The cost to redecorate each unit is $3,750.

Reconstruct the statement on the previous page and develop it into a proper income and expense statement and include a calculation of the operating expense ratio.

Review Questions

1. In addition to leases, _____ and _____ can be a valid source of market information, if care is exercised.

2. List five sources of rental income data:

   1. _____
   2. _____
   3. _____
   4. _____
   5. _____

3. Another name for market rent is _____ rent.

4. The actual rent paid in accordance with the terms of a lease is called _____ rent.

5. The fixed portion of the rent under the terms of a percent lease is known as _____ _____ _____ .

6. When contract rent exceeds market rent, the difference is known as _____ rent.

7. The rent that is over and above a guaranteed minimum base rent is referred to as _____ rent.

8. A contract that requires a fixed minimum rent and a variable rent base on volume of business, sales, productivity, or use of the property by the tenant is called a _____ lease.

9. A lease in which the landlord is required to pay all operating expenses associated with the real estate is known as a _____ lease.

10. A lease in which the tenant is required to pay some (or all) operating expenses associated with the real estate is called a _____ lease.

11. List four factors that must be considered when comparing rental properties:

    1. _____
    2. _____
    3. _____
    4. _____

12. List five common units of comparison when comparing rental properties:

    1. _____
    2. _____
    3. _____
    4. _____
    5. _____

13. The total leasable area, including common areas is known as _____.

14. The _____ is the potential gross income, less vacancy and collection loss, plus appropriate miscellaneous income.

15. Income from vending machines and coin-operated laundries is considered examples of _____ income.

16. The first step in reconstructing the income and expense statement is to determine the _____ .

17. Which of the following would be considered a proper expense in the reconstructed operating expense statement?

a. Salaries

b. Management

c. Utilities

d. All of the above

18. Short-lived items such as carpet, drapes, and water heaters are known as _____.

19. When reconstructing the income / expense statement, depreciation and debt service would be considered a/an _____ expense.

20. The income remaining after developing effective gross income and allowing for operating expenses and reserves for replacement is known as _____.

Chapter 3

Objectives

Upon completion of this chapter, you will be able to:

- Understand the relationship between property income and property value, as expressed by a capitalization rate.
- Understand the components and the functions of the various types of capitalization rates.
- Recognize the formulas that express the relationship between income and value.
- Understand the uses of the IRV and VIM equations in the valuation of income-producing properties.
- Recognize the definition of overall capitalization rate.
- Recognize the practical methods of developing an overall capitalization rate.
- Demonstrate the need for consistency in deriving and applying rates and factors.
- Develop the overall property yield rate from the band of investment technique and extraction from the yield-change formula.

Introduction

This chapter provides the student with the fundamentals for developing appropriate capitalization rates. Introduced in this chapter is the development of the property's overall capitalization rate, effective tax rate, and the overall yield rate. In this chapter, the student will learn the comparability requirements for deriving overall capitalization rates from comparable properties along with all the ways available to develop an overall capitalization rate. The student will also learn from this chapter the difference in the overall capitalization rate and the overall yield rate.

Introduction to Relationship of Capitalization Rates and the Value Estimate

A. Interest rates, yield rates, capitalization rates, and gross income multipliers are all considered relationships between income and value. For example, a savings account of $100 earning an interest rate of 6% produces interest income of $6 annually. An overall capitalization rate of 10% shows the relationship between an annual income of $10,000 and market value of $100,000, and a gross income multiplier of 5 can be used to demonstrate the relationship between a gross income of $50,000 and market value of $250,000.

B. No unit of comparison from market information has validity unless it relates to the subject property in the same way that it relates to market comparables from which it was derived.

C. IRV and VIM equations

1. I = R x V
2. R = 1 / V
3. V = I / R

4. V = I x M
5. I = V / M
6. M = V / I

D. Income expressed as percentage of an unknown value

1. Income expressed as a percentage of an unknown value - net operating income is a certain percentage of value and that percentage is called the overall capitalization rate.

2. In appraising a property, divide the percentage (known as the overall capitalization rate) into the property's net operating income, to .obtain value. Mathematically, if income is 10% of value of the property, then dividing the income by 10% will indicate the total value of the property.

E. Income and capitalization rates

1. Income must match rate

2. Rate must match income

3. No exceptions to this rule

Overall Capitalization Rate (Ro) or (OAR)

A. Expresses the relationship between net operating income and market value of property (The overall capitalization rate represents the percentage that net operating income is in relation to total property value.)

B. Used in direct capitalization

C. The overall capitalization rate (Ro) is a weighted average of the mortgage capitalization rate (Rm) and the equity capitalization rate (Re)-

D. Theoretically composed of the 'return on' rate (or overall yield rate Yo or property discount rate) and a portion of the 'return of (or recapture rate) for the improvements. While all the component parts are included in the overall capitalization rate, there is not necessarily any separate identification of each of the rates included within the overall capitalization rate.

E. Does not distinguish between land rate and improvement rate components. The overall capitalization rate (R_0) is a weighted average of the land and improvement capitalization rates used in straight-line capitalization. While all the component parts land capitalization rate (R_L) and improvement capitalization rate (Rb) are included in the overall capitalization rate, there is not necessarily any separate identification of each of the rates included within the overall capitalization rate.

Six Methods of Developing an Overall Capitalization Rate (Ro)

A. There are six methods which can be used to derive an overall capitalization rate (Ro)

B. The six methods are:

1. Market comparisons
2. Band of investment (mortgage & equity components)
3. Net operating income ratio

4. Debt coverage ratio

5. Yield change techniques

6. Band of investment (land & improvement components)

C. Market comparison method

1. Uses IRV equation

2. Known components of the IRV equation are:

   a) Net operating income

   b) Property value

3. Net operating income divided by the property value equals overall capitalization rate (R_o)

4. When attempting to derive an overall capitalization rate (Ro) from market sales transactions for use in direct capitalization, it is necessary that the sales transactions be highly comparable to the subject property. The disadvantage to using direct capitalization is that frequently the necessary data and comparability of properties are not available. If any significant adjustments must be made, then the method becomes inapplicable. The requirements for comparability include:

   a) Comparable types of property, with the same remaining economic lives, operating expense ratios, physical condition, and ratios of land-to-improvements as proportions of total property value.

   b) Comparable income streams with the same characteristics of risk, timing, stability, and income projection pattern.

   c) Comparable terms and types of financing.

   d) Comparable types of buyers with buying

motivations the same as those of the most probable type of buyer.

e) Comparable terms of sale.

f) Comparable market conditions at the time of sale and the time of appraisal.

g) Unless these conditions can be met, then the overall capitalization rate cannot correctly be estimated directly from market sales transactions.

5. Illustration of points of comparability - critical that the sale properties are highly comparable.

a) Land-to-improvement (building) ratios

b) Income and expense ratios

c) Remaining economic lives (improvements)

D. Band-of-investment method (weighted average) – mortgage and equity components:

1. Mortgage capitalization rate (R_M) or annual mortgage constant

a) The mortgage capitalization rate (R_M) is also known as the annual mortgage constant. It is different from the mortgage yield rate (Y_M).

b) The mortgage yield rate (Y_M) is the lenders return on the money borrowed, while the mortgage capitalization rate (R_M) is the ratio of the annual principal and interest payments to the amount borrowed.

c) Stated another way, the mortgage capitalization rate (R_M) is the percentage of the original loan that is required to be paid (principal and interest) annually.

2.

Equity capitalization rate (R_E) or equity dividend rate

a) The equity capitalization rate (R_E) is the ratio of the annual equity income to total equity investment.

b) Stated another way, the equity capitalization rate (R_E) is the percentage of the original equity investment that is represented by the income available to the original equity investment.

3. The mortgage capitalization rate (R_M) is different from the mortgage yield rate (Y_M). See compound interest tables in this text to show that the partial payment factor in column 6 (for annual compounding or column 6 times the number of compounding periods in one year for all other compounding) is the mortgage capitalization rate (R_M). Stated another way, the mortgage capitalization rate (R_M) is the ratio of the total mortgage payments for the year divided by the amount of money borrowed.

E. Net operating income ratio method

1. Based on dividing the net operating income ratio by the effective gross income multiplier (EGIM)

2. The net operating income ratio is 100 % minus the operating expense ratio.

F. Debt coverage ratio method

1 . Is often used by lending institutions that are sensitive to the safety of their loan investments. The lenders are concerned with profits expected by the property and providing a safety margin so that the borrower will be able to meet debt service obligations on the amount loaned on the property.

2. Formula is simple to use: $R_O = DCR \times M \times R_M$ where DCR equals the debt coverage ratio, M equals the loan-to-value ratio, and R_M equals the mortgage capitalization

rate.

3. Formula for developing the debt coverage ratio (DCR) is:

   a) NOI = Net Operating Income

   b) I_M = Income necessary to satisfy mortgage payments (annual debt service) [can also be computed by multiplying R_M times the amount borrowed].

G. Yield change techniques (within direct capitalization)

1 . Yield techniques using the general yield and change formula:

$$R_O - Y_O - CR$$

   a) R_O = Overall capitalization rate

   b) Y_O = Overall yield rate (also known as the property discount rate)

   c) CR = Constant rate of change in income and property value

2. Implicit in the use of this formula is the fact that both the income and property value are changing at the same compound rate and that the terminal capitalization rate (R_N) and the R_O are the same.

H. Band of investment method using land & building components

1. Must know the land capitalization rate (Rt) and building capitalization rate (Re)

2. Computed weighted average of land and building rates

3. The respective land and building rates are weighted by the percentage of value represented by land and building

respectively.

Effective Tax Rate

A. The percentage that annual real estate taxes represent in relation to total property value.

B. The effective tax rate can be computed by two methods - the EAT formula and market comparison.

Methods of Developing the Effective Tax Rate

A. EAT equation

1. E = the effective tax rate

2. A = the assessment level

3. T = the tax rate

B. Market comparison - like a recapture rate, the effective tax rate can be derived from market sales transactions.

1. Uses IRV equation

2. Known components of the IRV equation are:
   a) Real estate taxes
   b) Property value

3. Real taxes divided by the property value equals the effective tax rate.

Yield Capitalization

A. Yield capitalization is used to convert future income into present value by discounting each year's income at an appropriate yield rate or by applying an overall rate (extracted using one of the yield methods) that reflects the investment's income pattern, change in value, and yield rate.

B. Development of the property's overall yield rate (Y_o) - (also known as the property's discount rate or property's internal rate of return).

1. Can be developed by survey and opinion data from owners of similar property
2. Can be extracted from comparable sales
3. Can be derived from alternative investments of comparable risk
4. Can be developed by band of investment technique using mortgage interest rate and equity yield rate

   a) The mortgage interest rate (Y_M) is also known as the mortgage yield rate. It is different from the mortgage capitalization rate (K_M)- The mortgage yield rate (Y_M) is the lenders return on the money borrowed, while the mortgage capitalization rate (R_M) is the ratio of the annual principal and interest payments to the amount borrowed.

      (1) The mortgage yield rate (Y_M) can be developed by extraction from loans of similar risk.

      (2) The mortgage yield rate (Y_M) can be extracted from surveys of lenders in the market for the same level of risk.

   b) The equity yield rate (Y_E) is the rate of return on the equity capital invested. It takes into consideration all of the expected cash flows attributable to the equity investment, including proceeds from sale at the termination of the investment. The equity yield rate (Ye) can be developed by:

      (1) Extraction from comparable sales

      (2) Survey & opinion of market participants

(3) Comparison with the equity yield rates (Y_E) achieved in alternative investments of comparable risk such as stocks and bonds. While the equity yield rate (Y_E) can be developed from alternative investments of comparable risks such as stocks and bonds, the equity capitalization rate (R_E) used in direct capitalization cannot be developed correctly from earnings-price ratios of common stocks. See Significant comparisons in Example 3-2.

C. Factors included in overall yield rate (Yo)

1. Safe Rate
2. Risk Rate
3. Non – Liquidity Rate
4. Management Rate

Example 3-1

Development of an Overall Capitalization Rate (R_o) – Market Comparison Method

You are appraising a retail commercial property. In an effort to derive a proper overall capitalization rate (R_o) from market sales, you have found a similar retail commercial property, which recently sold for $900,000 and has an effective gross income of $200,000. Total operating expenses, including reserves for replacement, for the comparable sale amount to $96,500 annually. What is the indicated overall capitalization rate (Ro) for the comparable sale property?

Effective Gross Income	$200,000
Total Operating Expenses & Reserves	96,500
Net Operating Income	**$103,500**

Overall Capitalization Rate (R_o):

Net Operating Income	**$103,500**
Property Value	**$900,000**

Overall Capitalization Rate (R_o) = 11.5%

Example 3-2

Development of an (R_o) – by Market Comparison Method

Significant Comparisons

The following example demonstrates that even though two seemingly comparable properties may have the same sales price, the ratio of land-to-improvement (building) will affect the overall rate of the property even when the discount rate and the recapture rate are the same.

Item	Sale #1	Sale #2
Sale Price	800,000	800,000
Land Value	400,000	200,000
Improvement Value	400,000	600,000
Property Discount Rate (Yo)	0.10	0.10
Recapture Rate (20 Years)	0.05	0.05
Return for Land & Improvement	80,000	80,000
Recapture dollars (Depreciation)	20,000	30,000
Total Net Operating Income	100,000	110,000
Overall Capitalization Rate	0.125	0.1375

This example demonstrates the need for overall rates to be selected from comparable sales of improved properties having comparable land-to-improvement (building) ratios.

Example 3-2

Development of an (R_0) – by Market Comparison Method

Significant Comparisons (Continued)

In addition to the need for land-to-improvement (building) ratios to be comparable, the income and expense ratios of the comparable sales must be similar to ratios for the subject property being appraised. The following example demonstrates how two sale properties with comparable gross incomes and resulting identical gross income multipliers can have significantly different overall capitalization rates, due to differences in expense ratios.

Item	Sale #1	Sale #2
Gross Income	$200,000	$200,000
Expenses	$60,000	$80,000
Expense Ratio	30%	40%
Net Operating Income	$140,000	$120,000
Sale Price	$1,240,000	$1,240,000
Gross Income Multiplier	6.2	6.2
Overall Capitalization Rate	11.3%	9.7%

Example 3-2

Development of an (R_0) – by Market Comparison Method

Significant Comparisons (Continued)

Another requirement which must be met by the comparable sales properties and the subject property is they must have similar remaining economic lives. The example below shows how a change in the recapture rate (reciprocal of remaining economic life) can affect the overall rate.

Property Components (Assuming 80% Improvement and 20% Land)		
Item	**Sale #1**	**Sale #2**
Estimated Remaining Economic Life	25 Years	50 Years
Annual Recapture Rate	4.0%	2.0%
Property Discount Rate (Yo)	9.0%	9.0%
Improvement Cap Rate (Y_B)	13.0%	11.0%
Improvement Component of Overall Cap Rate	10.4%	8.8%
Land Component of Overall Cap Rate	1.8%	1.8%
Overall Cap Rate (R_o)	12.2%	10.6%

Sale #1

Capital Investment	Investment %	Cap Rate	Product
Land	0.20	0.09	0.018
Improvement	0.80	0.13	0.104
Totals	1.00		0.122

Sale #2

Capital Investment	Investment %	Cap Rate	Product
Land	0.20	0.09	0.018
Improvement	0.80	0.11	0.088
Totals	1.00		0.106

Example 3-2

Development of an (R_0) – by Market Comparison Method

Significant Comparisons (Continued)

From the preceding examples, we can see that the derivation of overall rates from properties that are not **very similar** to the subject can result in an invalid overall rate when applied to the subject. Derivation of overall rates from comparable sales requires highly comparable data. It is imperative that the properties used as comparables be comparable to the subject in terms of physical, locational, financial, and investment characteristics.

Exercise 3-1

Development of an R_O – Market Comparison Method

You are appraising an apartment complex and want to develop an overall rate from the market. You find six sales of similar apartment complexes. The following is the data for the six comparables:

Sale No.	Sale Price	Remaining Economic Life	Land-to-Improvement Ratio	Expense Ratio	Net Operating Income
1	760,000	18	1 : 4	34%	$64,600
2	780,000	20	1 : 5	27%	66,300
3	720,000	20	1 : 4	30%	64,800
4	712,000	20	1 : 4	30%	64,000
5	744,000	20	1 : 4	30%	67,000
6	750,000	22	1 : 5	30%	82,500

The subject property has a remaining economic life of 20 years, a land-to-improvement ratio of 1:4 and an expense ratio of 30%. The six comparables and the subject property are of similar construction and condition.

What is the indicated overall capitalization rate by market comparison, based on the six comparable sales?

Exercise 3-2

Development of an R_O – Market Comparison Method

You have been asked to appraise an apartment complex in your small town. You have only been able to find three sales of similar properties to the subject, all of which have remaining economic lives of 25 years. The subject is a 25 unit complex that has a 44% operating expense ratio and a remaining economic life of 25 years. The land-to-improvement ratio for the subject is 1:4. From the three comparable sales shown in the grid below, derive the overall capitalization rates for each and select the one that you believe is most appropriate for appraising the subject property.

	Comparable Sales		
Item	Sale #1	Sale #2	Sale #3
Sale Price	$2,100,000	$2,025,000	$2,550,000
No. of Units	25	24	30
Potential Gross Income	$410,000	$360,000	$450,000
V & C Loss	18,450	16,200	20,250
Operating Expenses	187,944	151,272	171,900
Land Value	$420,000	$405,000	$637,500

Exercise 3-3

Development of an Ro – Market Comparison Method

You are deriving an overall capitalization rate (Ro) from a market transaction which occurred recently for a price of $18,000,000. You have been given access to the owner's operating statement as shown below. All expenses appear to be reasonable, requiring only exclusion of improper expenses. In order to derive the overall capitalization rate (Ro) from the comparable sale, you must reconstruct the owner's operating statement and develop the appropriate information from which to derive the overall capitalization rate (Ro).

Owner's Operating Statement	
Gross Income	$3,450,245
Vacancy & Collection Loss	172,512
Effective Gross Income	$3,277,733
Expenses:	
Maintenance	393,328
Water & Sewer	65,700
Pool Cleaning	15,500
Insurance	115,000
Contributions to Political Party	25,000
Interest Expense	370,000
Electricity	491,500
Natural Gas	398,000
Management Expense	196,000
Depreciaton	520,000
Federal Income Taxes	378,000
Total Expenses	$2,968,028
Net Operating Income	$309,705

Example 3-3

Development of an (R_0) – by the Band of Investment Method

The overall capitalization rate (Ro) is the weighted average of the mortgage annual constant (mortgage capitalization rate RM) and equity capitalization rate (Re), weighted by the respective proportions of total property value that each represents. For example, assume the typical mortgage financing requires an 8% loan with monthly payments for 20 years. Looking up the mortgage annual constant in the compound interest tables for this loan indicates a rate of 0.100373.* The equity capitalization rate (Re) is 12%. Also assume that the debt represents 60% of value and the equity represents 40% of value.

To determine the overall capitalization rate (Ro) using the band-of-investment method with debt and equity components proceed as follows:

Financial Components	Percent of Investment		Rate		Product
Debt	0.60	x	0.100373	=	0.060224
Equity	0.40	x	0.120000	=	0.048000
Totals	1.00		**Overall Rate (R_0)**	=	**0.108224**

* NOTE: The mortgage annual constant or mortgage capitalization rate (R_M) can be computed with a financial calculator or found in the compound interest tables. For example, in the compound interest tables the partial payment factor (column 6) for a 20-year loan at an 8% interest rate with monthly payments is 0.008364. Because this partial payment factor is for monthly compounding it must be multiplied by 12 to get the total annual constant of **0.100373**. This means that 12 times the monthly partial payment factor is the annual amount that must be paid each year on the loan to fully amortize it over a 20-year (240-month) period.

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* The debt annual constant (R_M) is the ratio of the total mortgage payments for the year divided by the amount of money borrowed.

Exercise 3-4

Development of an R_O – Band of Investment Method Using Mortgage and Equity Components

You have determined that typical properties, like the subject you are appraising, are financed with 75% debt and 25% equity. From discussions with lenders and investors you have found that typical financing terms for this type of property require 9.3% of the amount borrowed be paid annually to amortize the loan, and the typical equity dividend rate (or equity capitalization rate - R_E) would be 10% for the equity investor.

From this information develop the overall capitalization rate (R_O) applicable to the subject property. *(Round your answer to 3 decimal places.)*

Exercise 3-5

Development of an R_O – Band of Investment Method Using Mortgage and Equity Components

You have been asked to develop an overall capitalization rate (R_O) for use in a commercial appraisal. You believe that similar commercial properties are financed with 80% debt and 20% equity. From discussions with investors and lenders, you have determined investors require an equity capitalization rate (R_E) of 6% for properties such as the subject and lenders require an 8% yield on loans of this type of mortgage, financed for 25 years with monthly payments. From this information you know that the mortgage capitalization rate (R_M) is 0.092618. *[You determined this mortgage capitalization rate (R_M) by looking in the compound interest tables or using a financial calculator — discussed in the next Chapter]*

From this information develop the overall capitalization rate (R_O) applicable to the subject commercial property. *(Round your answer to 3 decimal places.)*

Example 3-4

Development of an (R_0) – Use of the Net Income Ratio Method

The overall capitalization rate (Ro) can be developed by dividing the net income ratio by the effective gross income multiplier. The net income ratio is the ratio of net operating income to effective gross income. The effective gross income multiplier is the total sales price divided by the annual effective gross income.

For example, assume the effective gross income for a commercial property is $234,000 and the operating expenses for similar properties typically amount to 40 % of effective gross income. Also assume the property sold recently for $1,123,200. What is the overall capitalization rate?

The formula for determining the overall capitalization rate (Ro) using the net income ratio method is:

$$\mathbf{R_o} = \frac{\text{NIR (Net Income Ratio)}}{\text{EGIM (Effective Gross Income Multiplier)}}$$

NIR = Net Income Ratio
EGIM = Effective Gross Income Multiplier
(sometimes called a gross rent multiplier)

The net income ratio is 0.60 which is obtained by deducting the operating expense ratio of 40% from 100%. The effective gross income multiplier of 4.80 is developed by dividing the sales price of $1,123,200 effective gross income of $234,000. With this information the overall capitalization rate (Ro) can be computed as follows:

$$\mathbf{R_o} = \frac{0.60}{4.80} = 0.125 \text{ or } 12.5\%$$

Exercise 3-6

Development of an (R_0) – Use of the Net Income Ratio Method

Using the following market data, answer questions A through G.

A 10-unit apartment complex is receiving market rents of $1,500 per month. Vacancy and collection losses are projected to be 6%. Annual total operating expenses are forecast to be $53,721. The property recently sold for $ 1,100,000.

A. What is the potential gross income?

B. What is the effective gross income?

C. What is the net operating income?

D. What is the operating expense ratio?

E. What is the net income ratio?

F. What is the effective gross income multiplier?

G. What is the overall capitalization rate?

Exercise 3-7

Development of an (R_0) – Use of the Net Income Ratio Method

Develop the overall capitalization rate (R_o) from the following information which you have developed from sales comparables in the area of your subject property.

Potential Gross Income	=	$1,545,000
Vacancy and Collection Loss Rate	=	5%
Operating Expenses	=	$543,068
Effective Gross Income Multiplier (EGIM)	=	7.0

Example 3-5

Development of an (R_0) – Use of the Debt Coverage Ratio Method

In addition to other methods of calculating an overall rate for use in direct capitalization, another method based on the lending terms applicable to the property is available for use by the appraiser. This method (the debt coverage ratio method) is often used by lending institutions primarily concerned with the ability of the property to generate enough income to pay the annual debt service on the amount loaned on the property. The debt coverage ratio (DCR) is the ratio of annual net operating income (NOI) to the annual debt service (I_M), the income necessary to pay the annual amount of the mortgage principal and interest.

Formula: **DCR** = NOI / I_M (Annual Debt Service)

The overall capitalization rate (Ro) can be found by multiplying the debt coverage ratio times the annual mortgage constant and the loan-to-value ratio.

Formula: $\mathbf{R_o}$ = DCR x R_M x M

where:

R_o	=	Overall Rate
DCR	=	Debt Coverage Ratio
R_M	=	Annual Mortgage Constant (or Mortgage Cap Rate)
M	=	Loan-to-Value Ratio

To illustrate, assume that a property's net operating income is $700,000 and the annual debt service is $511,740. The debt coverage ratio is calculated as follows:

DCR = $700,000 / $511,740 = **1.3679**

If a mortgage can be obtained to finance 75% of the value of the property with an annual mortgage requirement of 11.19% of the amount financed, the overall rate is calculated as follows:

$\mathbf{R_0}$= 1.3679x0.1119 x 0.75
$\mathbf{R_o}$= 0.1148
$\mathbf{R_0}$ = 0.115 (rounded)

Exercise 3-8

Development of an (R_0) – Use of the Debt Coverage Ratio Method

You have determined that a commercial property can expect a net operating income of $930,000 from an analysis of market information. Further, it is expected that the annual debt service for the property will be $620,000. You have determined that 75% of the property's value can be financed by debt with an annual mortgage requirement of 8% of the amount financed.

What is the overall capitalization rate applicable to this property?

Exercise 3-9

Development of an (R_0) – Use of the Debt Coverage Ratio Method

Based on research and market analysis, you have learned that mortgage lenders in your area require a debt coverage ratio of 1.4 for the particular type of commercial property you are asked to appraise. The lenders have noted they will typically finance 60% of the investment for a term of 20 years in these commercial ventures, and the lenders require a 10% yield on their loans, which require level monthly payments of principal and interest. By using a financial calculator, you have determined that a 10% loan for 20 years with level monthly payments indicates a mortgage annual constant (or mortgage capitalization rate - R_M) of 11.58%.

Based upon the above data, what would be the indicated overall capitalization rate (R_o) indicated for the subject commercial property?

Example 3-6

Development of an (R_0) – Yield Change Formula

The overall capitalization rate (R_o) can be developed from the yield change formula. This methodology converts the property's overall yield rate (Yo) - (also known as the property's discount rate) to an overall capitalization rate (Ro)- The overall capitalization rate by the yield change formula is as follows:

Formula: $\mathbf{R_o = Y_o - CR}$

where:

Y_o = Property's overall yield rate
CR = Constant rate of change

Implicit in this formula is the requirement that both income and value are changing at the same compound rate (CR). Similar to the IRV equation, when any two components of the formula are known the third can be found. The three variations of the formula are:

Formula:
$$\mathbf{R_o = Y_o - CR}$$
$$\mathbf{Y_o = R_o + CR}$$
$$\mathbf{CR = Y_o - R_o}$$

The use of this formula, when these conditions occur, opens several possibilities to the appraiser. The appraiser can extract an overall capitalization rate (R_o) if the property's overall yield rate (Y_o) and the constant rate of change (CR) are known. Alternatively, if a property's overall capitalization rate (R_o) and constant rate of change (CR) is known, the overall yield rate (Y_o) can be extracted. Finally, if the property's overall capitalization rate (R_o) and overall yield rate (Y_o) are known, the constant rate of change can be extracted from the data.

Suppose an investor indicates he purchased a commercial property for which he required a 13% overall yield rate (Y_o). Further, he informs you that he expected the property's income and value to increase of a constant rate (CR) of change of 2% per year for the next 5 years. Based upon this information you may extract the overall capitalization rate from this investor's purchase. Solving for Ro produces the following results:

Formula: **Ro = Yo – CR**
Ro = 0.13 - .02 **Ro = 0.11 or 11%**

Example 3-6

Development of an (R_0) – Yield Change Formula – (continued)

Alternatively, suppose that a property sold for $720,000 with an expected net operating income (NOI) of $64,800. Additionally, the buyer's expected constant rate of change in net operating income and value was 3%. From this information, the appraiser can extract the property's overall yield rate. Solving for Y_o produces the following results:

Because we know the sales price and the expected net operating income we can extract the overall capitalization rate as follows:

Formula: R_o = NOI / Sale Price
R_o = $64,800 / $720,000 = 0.09 or 9%

Next by substitution, we can solve for the overall rate (discount rate) using the yield change formula.

Formula: Y_o = Ro + CR
Y_o = 0.09 + 0.03
Y_o = 0.12 or 12%

Finally, we can also solve for the constant rate of change (CR) if the other two components are known. Assume a commercial property sold for $1,200,000 with an expected net operating income (NOI) of $96,000. The buyer required a 10% overall yield rate on the purchase and expected the NOI and value to increase at a constant rate of change over the next five years. What is the expected constant rate of change (CR)?

Because we know the sales price and the expected net operating income, we can extract the overall capitalization rate as follows:

Formula: R_o = NOI / Sale Price
R_o = $96,000 / $1,200,000
R_o = 0.08 or 8%

Next, by substitution, we can solve for the constant rate of change (CR) using the yield change formula: **CR = Yo – Ro**

CR = 0.10 – 0.08
CR = 0.02 or 2%

Exercise 3-10

Development of an (R_0) – Yield Change Formula

You are gathering market data information to appraise a strip mall in your area. After discussions with investors in local strip malls, you learned that the overall property yield (Y_o) requirement for this type of investment is 15%. Further, you have determined that typically, similar strip malls expect to have their income and value increase at a constant rate of 3% annually over the next 5 years.

What overall capitalization rate (R_o) can be extracted from this information?

Exercise 3-11

Development of an (R_0) – Yield Change Formula

Your assignment is to estimate an overall capitalization rate (R_o) for a commercial office property in your township. Market research indicates owners and investors of this type of property expect income and value to grow at a constant rate of 2% per year for the next five years. You are going to appraise the property to yield 12.5%, the overall yield rate (Y_o) that typical owners and investors require at this time.

What overall capitalization rate (R_o) will you use in your appraisal?

Exercise 3-12

Development of an Overall Capitalization Rate (R_0) – Yield Change Formula

A property recently sold for $800,000, with an expected annual net operating income of $64,000 during the first year of ownership. The buyer expected an increase in income and value of 3% per year over the next 5 years.

What overall yield rate (Y_o) can be extracted from this sale?

Example 3-7

Development of an Overall Capitalization Rate (R_0) – Band of Investment Technique Using Land & Building Components

The overall capitalization rate (R_o) is the weighted average of the land and improvement capitalization rates, weighted by the respective proportions of total property value that each represents. For example, assume the land capitalization rate (R_L) is 10% and the improvement capitalization rate (R_B) is 14%. Assume further that the land represents 25% of value and the improvements represent 75% of value.

To determine the overall capitalization rate (Ro) using the band-of-investment method, proceed as follows:

Property Component	Percent of Investment		Cap Rate		Product
Land	0.25	x	0.1000	=	0.0250
Improvement	0.75	x	0.1400	=	0.1050
Totals	1.00				0.1300

Exercise 3-13

Development of an Overall Capitalization Rate (R_0) – Band of Investment Technique
Using Land & Building Components

You are developing overall capitalization rates for retail stores as a part of your mass appraisal process. You have determined that typical retail developments similar to the stores you are appraising are made up of land and building components of 30% and 70%, respectively.

From market extraction techniques, you have learned that the land and building capitalization rates for similar properties are 8% and 12%, respectively.

Determine the overall capitalization rate using the band-of-investment method (weighted average of land and building capitalization rates).

Exercise 3-14

Development of an Overall Capitalization Rate (R_0) – By Various Methods

Given the following information:

Sales Price	$6,000,000
Land Value	$1,250,000
First Mortgage (60% of total value)	60%
Equity Cap. Rate	12.50%
Net Operating Income	$660,000
Annual Mortgage Constant (R_M)	10%
Effective Gross Income	$1,200,000
Operating Expense Ratio	45%

Compute the overall capitalization rate (R_o) by using:

A. Market comparison (IRV)
B. Debt coverage ratio method
C. Net income ratio method
D. Band of investment method (mortgage & equity components)

Example 3-8

Determining the Effective Tax Rate by the EAT Formula

Level of Assessment

The level of assessment is the ratio of assessed value to full market value. In many jurisdictions, the assessed value (the taxable value of the property) is equal to the appraised value (full value or market value) of the property. In other jurisdictions, the assessed value is a fractional amount of the appraised value as set by state statutes. When the appraised value is the same as the assessed value, the level of assessment is 100% of the appraised value. When the assessment level is a fractional amount of the appraised value, the level of assessment is less than 100%. In Georgia the level of assessment is 40%.

Tax Rate

Tax rate structures vary from jurisdiction to jurisdiction. Some jurisdictions refer to the tax rate in terms of mill rate or dollars per thousand of assessed valuation, while other jurisdictions refer to the tax rate in terms of dollars per hundred of assessed valuation.

Tax rates, as dollars per hundred or as mill rates, can be converted to percentages. Dollars per hundred are converted to a percentage by dividing by 100. This accomplishes the same thing as moving the decimal point two places to the left. Mill rate is divided by 1,000. This accomplishes the same thing as moving the decimal point three places to the left. When applying the tax rate, the appraiser must always express the rate as a decimal.

The same tax rate can be expressed several ways. The following in an example of a 3% tax rate.

$30 per $1,000	(30 / 1,000 = 0.03)
$3 per $100	(3 / 100 = 0.03)
$.03 per $1	0.03 / 1 = 0.03)
30 mills per $1	(30 / 1,000 = 0.03)

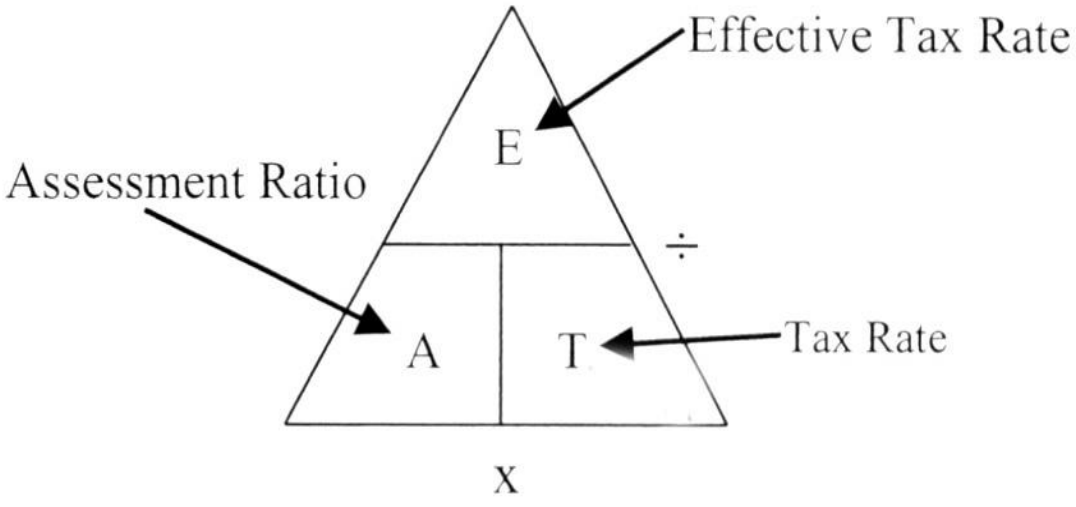
Example 3-8

Determining the Effective Tax Rate by the EAT Formula – (continued)

The income approach considers real estate taxes as a proper expense. However, the appraiser for ad valorem tax purposes is establishing value for this purpose. Therefore, it would be incorrect to deduct a predetermined amount for real estate tax based on a prior appraisal. Instead, the appraiser for ad valorem tax purposes should not deduct the current real estate tax liability from the operating income but instead develop an effective tax rate and include it as a component in the overall capitalization rate.

The effective tax rate is the ratio of annual real estate taxes to the total property value. When the level of assessment is 100% of appraised value or market value, the effective tax rate and the tax rate are same. Where assessments are less than 100% (fractional assessments) then the effective tax rate is the ratio of tax rate to assessment level.

To calculate the effective tax rate in jurisdictions with fractional assessments, the official tax rate is multiplied by the assessment level. (All figures must be expressed in proper decimal format.)



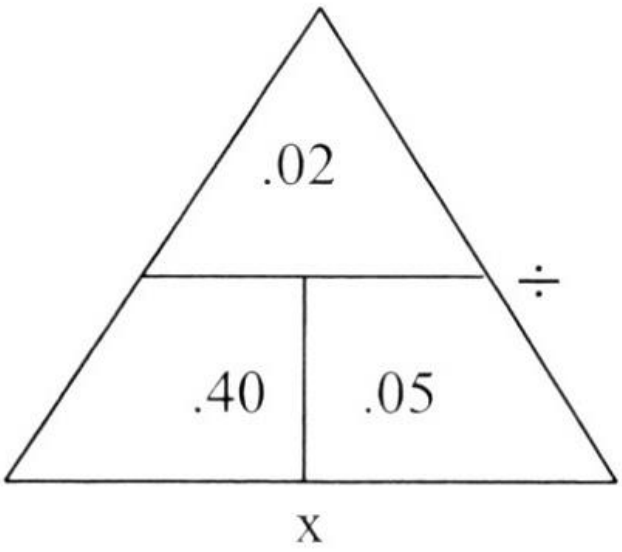
Example 3-8

Determining the Effective Tax Rate by the EAT Formula – (continued)

Example:

The assessment level is 40% of appraised value and the current tax rate is $5.00 per hundred. This provides and effective tax rate as follows:

Assessment level		0.40
Tax Rate	x	0.05
Effective Tax Rate		**0.02 or 2.0%**



Example 3-9

How Tax Rates are Expressed

No	Tax Rate	Decimal Equivalent
1	$65.00 / $1,000	
2	72 Mills	
3	$8.00 / $100	
4	$80.00 / $1,000	
5	115 Mills	
6	$4.50 / $100	
7	62.5 Mills	
8	$75.25 / $1,000	
9	$12.75 / $100	
10	$6.25 / $100	

Table: How Tax Rates Are Calculated

Exercise 3-15

Development of the Effective Tax Rate

1. Complete the following table of equivalencies:

Mills		$ Per $100		Nominal Tax Rate
	=	2.60	=	
35.00	=		=	
	=		=	0.0100
	=	7.25	=	0.0725
6.00	=		=	
	=		=	0.0150

2. Complete the following table. Use the EAT triangle formula.

Assessment Level	Nominal Tax Rate $ Per $100	Mills	Effective Tax Rate
0.25	5.50		
		60.00	0.03600
0.35			0.02975
0.50		62.50	
0.40		70.00	0.02800
	7.75		0.02325

Exercise 3-15

Development of the Effective Tax Rate – (continued)

Market Value	Tax Dollars	Effective Tax Rate
$560,000	$12,880	
$900,000		0.033
$680,000	$19,040	
	$32,500	0.025
$940,000		0.026
	$23,100	0.035

** Note Effective Tax Rate is Expressed as a Decimal

Example 3-10

Determining the Effective Tax Rate by Market Comparison

The effective tax rate can also be obtained when the dollar amount of the real estate taxes and the appraised or market value of the property is known. This is accomplished by dividing the dollar amount of real estate taxes by the appraised or market value of the property.

Example:

Current Taxes	$4,000
Property Value	$200,000
Effective Tax Rate	($4,000 / $200,000) = **0.02 or 2.0%**

Exercise 3-16

Development of the Effective Tax Rate by Market Comparison

A. Market Comparison

The Smith's property sold recently for $360,000. The assessment level in this area is 40%. The prior tax bill for the Smith's was $5,400. What is the effective tax rate?

B. EAT Formula

The Jackson's property is located in an area of town that was appraised last year. The current assessment level is 40%. The current property tax is based on 37.5 Mills. What is the effective tax rate?

Exercise 3-17

Development of the Effective Tax Rate by Market Comparison

You are developing effective tax rates for several tax jurisdictions. You have found the following sales with the taxes paid in the latest year for each sale.

Tax Jurisdiction: **North Side**

Sale A sold recently for $420,000 and had an annual tax bill of $7,560.
Sale B sold one month ago for $375,000. The annual taxes are $6,750.

Tax Jurisdiction: **South Side**

Sale C sold recently for $165,000 and had an annual tax bill of $2,063.
Sale D sold 18 days ago for $184,000. The annual property taxes were $2,300.

What were the effective tax rates for each sale?

Example 3-11

Deriving the Property's Overall Yield Rate (Y_o) by the Band of Investment Method

In certain situations the overall yield rate (Y_o) for a property can be developed by the band of investment technique of weighting the mortgage yield rate (Y_M) and equity yield rate (Y_E) by their respective percentages of the total financing for a property. In instances where the loan is "interest only" and there is no change in income or value over the investment holding period, the appraiser can use this technique to develop the property's overall yield rate (also known as the property's discount rate).

For example, assume 80% of a commercial property is to be financed with an 8% interest only mortgage with a balloon payment at maturity. Further, assume that the income and value for the property is not expected to change during the remaining economic life of the property and the equity investor requires a yield of 13%. The overall yield rate for the property can be calculated to be 9% as follows:

Capital	Portion		Yield Rate		Product
Debt	0.80	x	0.08	=	0.0640
Equity	0.20	x	0.13	=	0.0260
Totals			**Yield Rate (Y_o)**		**0.0900**

NOTE: Blending a Y_M and a Y_E to obtain a Y_o by the band of investment technique is mathematically incorrect unless the loan is interest only and there is no change in income or value over the projection period. The primary reason the equity yield rate is not correct is the fact that the loan-to-value ratio (M) is not the same throughout the holding period.

Exercise 3-18

Extracting the Property's Overall Yield Rate (Y_o) by the Band of Investment Method

You have been asked to derive the overall yield rate (Y_o) for a public utility property that is expected to be financed with interest only debt requiring a balloon payment at maturity to pay off the mortgage. You believe this is typical financing for utility properties of this type, because most appear to be financed with long-term bonds. Lenders have informed you that their yield requirement for this type of debt is 8% and you know that equity investors require a 12% yield for this level of risk.

Market information indicates that investors in this type of property typically finance 40% of this type of purchase with debt and 60% with equity. Further, you believe there will be no change in value or income during the holding period because of regulatory restrictions.

What is the indicated overall yield rate (Y_o) for this investment?

Example 3-12

Extracting the Property's Overall Yield Rate (Y_o) by Yield Change Formula

The overall yield rate (Y_o) can be extracted from sales using the same yield change formula used in computing the overall capitalization rate $R_o = Y_o - CR$, if the same requirements are met as in deriving the overall capitalization rate. Those requirements are that the property's income and value are expected to increase at the same rate over the holding period of the investment.

Consider the following: If a commercial property recently sold for $5,600,000. This property is expected to produce a net operating income of $476,000 during the first year of ownership. The buyer indicated an expected increase in income and value of 3.5% per year over the next 5 years.

What overall yield rate (Y_o) can be extracted from this sale?

The overall capitalization rate (R_o) is found to be 0.085 by dividing the net operating income by the sale price. Then by substitution in the formula: (See Example 3-6 also.)

$$\mathbf{Y_o = R_o + CR}$$
$$Y_o = 0.085 + 0.035$$
$$Y_o = 0.12$$

Exercise 3-19

Extracting the Property's Overall Yield Rate (Y_o) by Yield Change Formula

A commercial property recently sold for $3,500,000. This property is expected to produce a net operating income of $315,000 during the first year of ownership. The buyer indicated he expected an increase in income and value of 4% per year over the next 5 years.

What overall yield rate (Y_o) can be extracted from this sale?

Review Questions

1. Name the six methods of developing an overall capitalization rate (R_o).

   1. _____

   2. _____

   3. _____

   4. _____

   5. _____

   6. _____

2. The summation method of estimating an overall yield rate (Y_o) [property discount rate] is a theoretical method which is based upon the summation of four components that affect the return on or requirements of an investment. The four components are:

   1. _____

   2. _____

   3. _____

   4. _____

3. When developing the overall capitalization rate from market comparisons, three Significant items which must be highly comparable between comparable sales and the subject are:

   1. _____

   2. _____

   3. _____

4. Another name for the mortgage capitalization rate (R_M) is the

   _____ _____ _____.

5. The _____ ____ _____ method computes a weighted average of the mortgage capitalization rate (R_M) and equity capitalizations rate (R_E). The resulting weighted average is called the overall capitalization rate (R_O).

6. The lender's yield on a mortgage is known as the _____ _____ rate.

7. The _____ _____ _____ is the percentage that annual real estate taxes are in relation to the total property value.

8. Name the two methods of developing an effective tax rate.

   1. _____

   2. _____

9. The net income ratio method can be used to develop an overall capitalization rate when the net income ratio is divided by the _____ _____ _____

   _____.

10. In order to use the yield change technique to get an overall capitalization rate, the appraiser must know two of these three components:

    1. _____

    2. _____

    3. _____

11. The _____ _____ _____ for developing an overall capitalization rate is appropriate when the net operating income and value are expected to change at a constant rate over the holding period.

12. The _____ _____

    _____ reflects the relationship between the land income and the land value.

13. The weighted average of the land capitalization rate and the building capitalization rate is known as the _____ _____ _____.

14. Dividing the building (improvement) income by the _____ _____ _____ will result in the building (improvement) value.

15. The assessment level is 30% of appraised value and the current tax rate is $8.50 per hundred. What is the effective tax rate expressed as a decimal and a percentage? _____ or _____.

Quiz #1

1. Capitalization is the process used to:

   A. establish reproduction cost
   B. establish mortgage payments
   C. establish a depreciation schedule
   D. convert income into an estimate of value

2. A loan secured by real property featuring an interest rate that is constant for the term of the loan is referred to as a (an):

   A. adjustable-rate mortgage
   B. fixed-rate mortgage
   C. constant amortization mortgage
   D. non-adjustable-rate mortgage

3. The underlying principle providing the basis of the income capitalization approach is:

   A. change
   B. balance
   C. conformity
   D. anticipation

4. The basic equation used in the income approach to value is:

   A. rate divided by income equals value
   B. income divided by rate equals value
   C. rate times income equals value
   D. rate plus income equals value

5. A gross lease requires that in addition to the rent, the tenant must pay:

   A. insurance and taxes
   B. maintenance & utilities only
   C. maintenance, utilities, insurance, and taxes
   D. none of the above

6. From the tenant's standpoint, when economic rent exceeds contract rent the difference is known as:

   A. leased fee income
   B. leasehold income
   C. economic income
   D. sandwich interest income

7. The rental income that a property would most probably command in the open market is called:

   A. net rent
   B. gross rent
   C. market rent
   D. contract rent

8. The anticipated income from all operations of the property adjusted for vacancy and collection losses, and miscellaneous income is called:

   A. pre-tax income
   B. net operating income
   C. potential gross income
   D. effective gross income

9. If an income property has an annual effective gross income of $64,000 with total expenses of $30,000 and variable expenses of $17,000, what is the operating expense ratio?

   A. 0.47
   B. 0.27
   C. 0.73
   D. 2.13

10. In reconstructing an income statement for an apartment complex, you estimate that the potential gross income is $500,000. The vacancy and collection loss allowance is 6 percent. If operating expenses are $205,000, what is the operating expense ratio (rounded)?

   A. 41 percent
   B. 44 percent
   C. 45 percent
   D. Operating expense ratio cannot be determined without knowing the amount of the mortgage payment.

Questions 11, 12, 13, and 14 are based on the following information.

The subject is an office building which contains 30,000 square feet of gross building area, and 24,000 square feet of net rentable area. The market rent for similar space is $ 15.00 per square foot of net rentable area. Vacancy and collection losses are 3% of gross potential income. Operating expenses are 32% of effective gross income, and the sales price was $2,793,600.

11. What is the potential gross income?

   A. $349,200
   B. $360,000
   C. $436,500
   D. $450,000

12. What is the effective gross income?

A. $349,200
B. $360,000
C. $370,800
D. $436,500

13. What is the net operating income?

A. $115,200
B. $139,680
C. $237,456
D. $306,000

14. What is the overall capitalization rate (R_o)?

A. 0.085
B. 0.110
C. 0.105
D. 0.075

15. A property has a net operating income of $10,000, interest payments of $8,000 and principal payments of $ 1,000. What is the debt coverage ratio?

A. 0.09
B. 0.10
C. 1.11
D. 0.08

16. Given the following information:

    Building capitalization rate: 0.11
    Land capitalization rate: 0.09
    Land value as a percent of total value: 35 percent

    What is the overall capitalization rate by using the band-of-investment method?

    A. 0.097
    B. 0.100
    C. 0.103
    D. 0.110

17. An apartment building is valued at $420,000 and has a net income of $2,800 per month. Calculate the overall capitalization rate for the investment.

    A. 6.67%
    B. 7.52%
    C. 8.00%
    D. 12.50%

18. Which of the following calculations is <u>not</u> needed to use the band-of-investment method?

    A. reversion
    B. loan-to-value ratio
    C. rate for equity
    D. rate for debt

19. The rate that is the percentage that annual real estate taxes are in relation to the property's total value is:

    A. effective tax rate
    B. after-tax rate
    C. recapture rate
    D. nominal interest rate

20. The ratio of net operating income to effective gross income is called:

    A. land capitalization rate
    B. net income ratio
    C. operating expense ratio
    D. .effective gross income ratio

21. The mortgage capitalization rate (R_M) is the percentage of the original loan that is required to be paid:

    A. Monthly
    B. Annually
    C. Quarterly
    D. By the lender

22. What is the overall capitalization rate (R_o) from the following information which you have developed from sales comparables in the area of your subject property?

    Potential gross income = $1,545,000; Vacancy and collection loss rate = 5%; Operating expenses = $543,068; Effective gross income multiplier = 7.0

    A. 0.08
    B. 0.09
    C. 0.10
    D. 0.11

23. You have determined that typical properties, like the subject you are appraising, are financed with 60% debt and 40% equity. From discussions with lenders and investors you have found that typical financing terms for this type of property require 8.0% of the amount borrowed be paid annually to amortize the loan, and the typical equity dividend rate (or equity capitalization rate (R_E) would be 11 for the equity investor.

    From this information develop the overall capitalization rate (Ro) applicable to the subject property. *(Round your answer to 3 decimal places.)*

    A. 0.087
    B. 0.065
    C. 0.092
    D. 0.110

24. A commercial property recently sold for $7,000,000. This property is expected to produce a net operating income of $630,000 during the first year of ownership. The buyer indicated he expected an increase in income and value of 4% per year over the next 5 years. What overall yield rate (Y_o) can be extracted from this sale?

    A. 0.13
    B. 0.12
    C. 0.11
    D. 0.14

25. The debt coverage ratio requirement from mortgage lenders is 1.5. Lenders will finance 80% of the investment and require an annual mortgage constant of 11.5%. What would be the indicated overall capitalization rate (R_o) indicated for the subject commercial property?

    A. 10.6%
    B. 15.2%
    C. 12.5%
    D. 13.8%

Chapter 4

Objectives

- Understand and identify the definition of direct capitalization
- Identify the mathematical relationships between income and value used in direct capitalization and explain how they are indicated in sales transactions and other market evidence.
- Understand the derivation and application of gross income multipliers in direct capitalization.
- Identify the common symbols used in the income capitalization approach.
- Recognize the procedures used to process income estimates and convert income to value using direct capitalization.
- Understand and identify the definition of yield capitalization.
- Explain the differences between yield capitalization and direct capitalization.
- Identify and understand the definition of an annuity.
- Identify the income stream patterns used in yield capitalization.
- Understand and identify the definition and application of discounting.
- Understand and identify the definition of discounted cash flow (DCF) analysis and the components needed for application in yield capitalization.
- Identify the six basic financial functions and the formulas used to compute each financial factor.
- Understand how the compound interest tables can be used for solving common appraisal problems.
- Explain the difference between an effective interest rate and a nominal

interest rate.

Introduction

Contemporary Capitalization Methods is designed to teach the students current capitalization procedures that are appropriate in modern appraisal problems and when it is appropriate to use direct capitalization and yield capitalization. This chapter demonstrates the use of direct capitalization using both Overall Capitalization Rates and Gross Income Multipliers.

Also, the student is introduced to yield capitalization and the requirements for using yield capitalization. As part of the yield capitalization techniques the student is introduced to compound interest tables and the six functions therein in order to estimate the market value of level and variable income streams with reversions
and the requirements for using yield capitalization. As part of the yield capitalization techniques the student is introduced to compound interest tables and the six functions therein in order to estimate the market value of level and variable income streams with reversions.

Capitalization Methods

A. Direct capitalization - a method of converting an estimate of a single year's income into value in one direct step. This conversion is made in one direct step, either by dividing the income estimate by an appropriate rate or by multiplying the income by an appropriate income factor.

B. Direct capitalization is different from yield capitalization because direct capitalization does not directly consider the individual cash flows beyond the first year. However, yield capitalization explicitly calculates year-by-year effects of each year's cash flow, changes in income patterns, changes in the original investment's value, and other considerations.

C. Either direct capitalization or yield capitalization will produce a proper indication of value when correctly applied. For direct capitalization to be applied correctly, it must be based upon relevant market information derived from comparable properties, which should have similar income-expense ratios, land value-to-building value ratios, risk characteristics, and

future expectations of income and value changes over a typical holding period. A choice of capitalization methods (yield or direct) will not produce a different value indication under this circumstance.

D. Characteristics of direct capitalization

1. Converts a single year's income estimate into value indication

2. No distinction between return of and return on investment

3. Investor assumptions or forecasts of the holding period, pattern of income, or changes in the value of the original investment are not identified.

4. Direct capitalization may be used on various income levels some of which are: potential gross income, effective gross income, net operating income, land income, and improvement income.

5. The generic formulas used in direct capitalization are:

   a) Value = Income / Rate

   b) Value = Income x Factor

E. Yield capitalization - a method of converting future net benefits into present value where each future net benefit is discounted at a proper yield rate (discount rate). Yield capitalization can also be accomplished by developing an overall rate that specifically reflects the investment's pattern of income, change in value, and yield rate.

F. Steps in yield capitalization

1. Select the proper holding period.

2. Estimate all future net benefits.

3. Select the appropriate yield (discount) rate.

4. Convert all future net benefits into present value by discounting each future net benefit or by developing a proper overall rate which reflects the pattern of income,

change in value, and yield rate.

G. Application of the discounting procedure requires the appraiser to be familiar with patterns of income, typical holding periods, discounting calculations, anticipated change in value, and discount rates relative to the perceived risk of the investment.

Application of Direct Capitalization

A. Overall capitalization rate (R_o)

1. Without property tax component

   a) When property is valued for ad valorem tax purposes, property taxes cannot be shown as an operating expense because the actual taxes are not known as of the assessment date. Indeed, the appraisal is often done to estimate the amount of tax.

   b) The problem can be resolved by developing an effective tax rate (as shown in Chapter 3) and adding it to the overall capitalization rate (R_o) for the subject property.

2. With a property tax component

3. The ad valorem tax appraiser must be aware of and correctly match the appropriate net operating income (which includes an amount to pay property taxes) with an overall capitalization rate (which includes an effective property tax rate component). Whatever income is in the numerator must also be included in the rate in the denominator. (See Chapter 3 regarding matching of numerator and denominator.)

4. Direct capitalization of land

   a) Required comparability criteria

      (1) Amenities - similar locations, restrictions, investor (economic) desirability, utility, etc.

      (2) Expense ratio - Even if the amenities are similar, if the expense ratios are different

for the comparables as compared to the subject, a proper overall rate cannot be derived.

b) Effective tax rate

(1) Included

(2) Excluded

5. Direct capitalization of improved properties

a) Required comparability criteria

(1) Amenities - similar locations, restrictions, investor (economic) desirability, utility, etc.

(2) Land-to-improvement (building) ratios

(3) Expense ratios

(4) Remaining economic life

b) Effective tax rate

(1) Included

(2) Excluded

B. Gross income multipliers (gross rent multipliers)

1. Residential properties

2. Commercial properties

a) Gross income multipliers (GIM) can be developed using either potential gross income or effective gross income. There are times when one is more appropriate than the other depending on the application. Either application is correct mathematically as long as the appraiser is consistent in deriving and using the GIM. Consistency in application here is important.

b) Normally, the gross income multiplier is based on effective gross income (EGIM) - just like

expense ratios are normally based on the relationship between effective gross income and total operating expenses. However, like expense ratios, it is mathematically correct to base expense ratios on potential gross income as long as the appraiser is consistent in the application of the ratios and multipliers.

Introduction to Yield Capitalization

A. Yield capitalization converts a series of future cash flows over time into an indication of value. Yield capitalization is used to convert the future cash flows into present value by applying an appropriate yield rate. The application of capitalization rates that reflect an appropriate yield rate, the use of present value factors, and discounted cash flow analysis are all yield capitalization procedures. The present value factors used in yield capitalization are derived from compound interest formulas and calculations. These formulas and calculations are the basis of the compound interest tables.

B. Compound interest tables

1. Expresses the time value of money - a dollar received today is worth more than a dollar to be received in the future

2. Effective interest rate vs. nominal interest rate

   a) The effective interest rate is the nominal interest rate divided by the number of conversion periods in one year. For example, if the nominal interest rate is 0.09, the effective interest rate would be 0.0075 for monthly compounding. (0.09 / 12 = 0.0075)

   b) The effective interest rate is used in all calculations for the compound interest tables.

3. Involves six interest functions which measure present worth and future worth

   a) Future worth of $ 1

(1) is the basis of all interest functions

(2) shows the growth of a single deposit over a specific time period

(3) is also known as the compound amount of $1

(4) is the reciprocal of the present worth of $ 1

b) Future worth of $ 1 per period

(1) shows the growth at compound interest of a series of deposits

(2) is also known as the compound amount of $1 per period

(3) is the reciprocal of the sinking fund factor

c) Sinking fund factor

(1) shows the amount of periodic payment necessary to accumulate to a specific sum

(2) is the reciprocal of the future worth of $ 1 per period

d) Present worth of $ 1

(1) shows the present worth of a single future payment

(2) is the reciprocal of the future worth of $1

e) Present worth of $ 1 per period

(1) shows the present worth of a series of future level income payments (an annuity)

(2) is also known as the annuity factor or the Inwood coefficient

(3) is the reciprocal of the partial payment factor

f) Partial payment factor

(1) shows the periodic payment necessary to amortize a loan at a specific interest rate over a specific period of time

(2) is also known as the installment to amortize $1

(3) is the reciprocal of the present worth of $1 per period

(4) The partial payment factor can be used to produce the mortgage annual constant, also known as the mortgage capitalization rate (R_M), when multiplied by the number of conversion periods in a year. For example, the partial payment factor is 0.009650 for a loan requiring monthly payments to be amortized over a 20 year period at an interest rate of 10%. This partial payment factor can be multiplied times the number of conversion periods in one year, 12 payments, to get the mortgage annual constant of 0.115803. In other words, 0.115803 multiplied times the amount of the loan will indicate how much principal and interest must be paid each year to fully amortize the loan over the 20-year period. The mortgage annual constant is used in direct capitalization to get an overall rate when multiplied by the percentage of the investment represented by debt and added to the product of the equity dividend rate, also known as the equity capitalization rate (R_E),

times the percentage of the investment represented by equity.

C. Discounted cash flow - This can be used to estimate the present value of any pattern of income or cash flow collections to be received in the future. The appraiser must identify the cash flows, determine the holding period, select an appropriate yield rate, and discount the cash flows to present value. Provided these four steps can be accurately accomplished, the appraiser should be able to render a very accurate estimate of market value. The appraiser can discount each individual cash flow separately indicating a present value for each payment received and add the present values to obtain an indication of market value.

1. Single future payment - To find the present value of a single payment to be received at a point in the future, one must select an appropriate yield rate and discount the payment into an indication of present value using the present worth of $1 factor (also known as the reversion factor). This process is commonly done when estimating the present value of the expected future reversion of a property at the end of a lease.

2. Annuities / rents

a) Level annuities / rents - The appraiser is often faced with estimating the value of a property subject to a lease. If the lease contract calls for level lease payments over the life of the lease, the lease has the characteristics of an ordinary annuity and the value of the property can be estimated using the present worth factors as discussed previously. A level annuity has an unchanging flow of income over time and the payments are regularly scheduled and made in accordance with a predetermined agreement. These problems are easily solved using discounted cash flow calculations. The cash flows discounted into present value can be the net operating income to the entire property or the cash flows to a specific interest such as cash flow to equity for the equity value or the cash flows to the mortgage for the value of the mortgage.

b) Variable annuities / rents - Often the appraiser must estimate the value of a property subject to a lease which has varying levels of income due to

adjustment clauses in a lease agreement or simply because price levels and operating expenses change. Thus, the appraiser must determine the value of an income stream which has no systematic payment schedule. To estimate the value of a variable annuity or a variable income stream, the appraiser must separately calculate the present value of each income payment in accordance with its expected collection. The separate values are then summed to obtain the total value of the series of cash flows.

D. When valuing a variable annuity using yield capitalization, it is not mathematically correct to add the effective tax rate (ETR) to the overall yield rate (Y_o) for obtaining an appropriate overall yield rate including a tax component. This is because the effective tax rate is a ratio between property taxes and value, while the overall yield rate conceptually is not a ratio between income and value. Even though an overall capitalization rate (R_o) can be adjusted by adding an effective tax rate [both are ratios between income and value], an overall yield rate (Y_o) cannot be correctly adjusted by simply adding or subtracting the effective tax rate. <u>This concept is beyond the scope of this basic income course and thus all cash flows discounted in the yield capitalization section of this course are conceptually after property taxes have been deducted.</u>

Example 4-1

Direct Capitalization of Land

You are appraising commercial properties in a core metropolitan area. Your current assignment is the appraisal of two parking lots. A search of sales data produced four recent sales of parking lots that will not require any adjustment for market conditions. The four sales were confirmed as arms-length transactions, sold with typical financing, and were leased at current market rent. Data on the four sales is as follows:

	Property A	Property B	Property C	Property D
Sale Price	$1,300,000	$1,200,000	$1,600,000	$1,100,000
Square Foot Area	21,258	22,027	31,934	21,875
Shape	Rectangular	Triangular	Irregular	Rectangular
Topography	Level	Level	Level	Level
Drainage	Adequate	Adequate	Good	Good
Operating Expense Ratio	10.0%	10.5%	12.5%	10.0%
Net Operating Income (NOI)	$126,000	$118,000	$143,000	$108,000
Effective Tax Rate	0.0115	0.0125	0.0100	0.0140

You need to develop an overall rate to apply to your subject's net operating incomes. However, since the four sales are located in tax areas that are different than the tax area for the subjects, the income attributable to taxes must be removed. This will provide an overall capitalization rate exclusive of a tax component. *For vacant land, an overall capitalization rate (R_o) without a tax component is equal to the land capitalization rate.*

Your next step is to remove the income to taxes and then calculate an overall rate for each comparable. This analysis produces the following:

	Property A	Property B	Property C	Property D
Sale Price	$1,300,000	$1,200,000	$1,600,000	$1,100,000
Square Foot Area	21,258	22,027	31,934	21,875
Shape	Rectangular	Triangular	Irregular	Rectangular
Topography	Level	Level	Level	Level
Drainage	Adequate	Adequate	Good	Good
Operating Expense Ratio	10.0%	10.5%	12.5%	10.0%
Net Operating Income (NOI)	$126,000	$118,000	$143,000	$108,000
Effective Tax Rate	0.0115	0.0125	0.0100	0.0140
Income To Taxes	$14,950	$15,000	$16,000	$15,400
NOI Without Taxes	$111,050	$103,000	$127,000	$92,600
Overall Rate (R_o)	0.08542	0.08583	0.07938	0.08418

Exercise 4-1

Direct Capitalization of Land

You are to appraise two parking lots that are in the same general location, and in the same tax area. You have found three recent sales not requiring any adjustment for market conditions. However, the comparable sales are not in the same tax area as the subjects. The three sales were confirmed as arms-length transactions, sold with typical financing, and were leased at current market rent. Data on the three sales is as follows:

	Sale One	Sale Two	Sale Three
Sale Price	$1,950,000	$1,800,000	$2,400,000
Square Foot Area	31,850	33,000	46,500
Shape	Irregular	Irregular	Rectangular
Topography	Level	Level	Level
Drainage	Adequate	Good	Good
Operating Expense Ratio	0.120	0.120	0.135
Net Operating Income (NOI)	$189,000	$177,000	$214,500
Effective Tax Rate	0.0175	0.0165	0.0155

Data on the subject properties is as follows:

	Subject A	Subject B
Square Foot Area	$32,400	$33,500
Shape	Irregular	Irregular
Topography	Level	Level
Drainage	Adequate	Good
Operating Expense Ratio	0.120	0.120
Net Operating Income (NOI)	$192,700	$198,400
Effective Tax Rate	0.020	0.020

Develop an overall capitalization rate that can be applied to the subject properties to obtain an estimate of value using direct capitalization.

Example 4-2

Direct Capitalization of Improved Property

Your appraisal assignment is an apartment complex containing five buildings with twelve units in each building. You have conducted a market analysis and established market rent for the subject. You have also reconstructed the income/expense statement, which provided a net operating income of $406,000 and an operating expense ratio of 39%. Land in this area represents 25% of a property's value. Your estimate of the remaining economic life for this property is 20 years. The effective tax rate for the subject is 0.020.

You have found three recent sales of similar apartment complexes that will not require any adjustment for market conditions. However, the three comparables are located in different tax areas. This requires you to develop an overall rate without a tax component, and then add the effective tax rate appropriate for the subject's location. The three sales were confirmed as arms-length transactions and sold with typical financing. A market analysis found the properties rented at market rent and you have reconstructed the income/expense statements for each comparable. Data on the three comparable sales is as follows:

	Sale 1	Sale 2	Sale 3
Sale Price	$5,760,000	$5,610,000	$5,900,000
Land / Building Ratio	1 : 3	1 : 3	1 : 4
Remaining Economic Life	22	20	20
Operating Expense Ratio	40%	39%	40%
Drainage	Adequate	Good	Good
Net Operating Income (NOI)	$414,000	$410,500	$424,800
Effective Tax Rate	0.018	0.0225	0.019
Income To Taxes	$103,680	$126,225	$112,100
NOI After Taxes	$310,320	$284,275	$312,700
Overall Rate (R_o)	0.0539	0.0507	0.0530

Now you must analyze the results obtained from the three comparables and select a rate most appropriate for the subject property. In selecting the overall rate, it is necessary to consider the similarities and differences between the subject and the comparables. The subjects' land-to-improvement ratio is the same as Sale 1 and Sale 2. The subjects' remaining economic life is the same as Sale 2 and Sale 3. The subjects' expense ratio is the same as Sale 2. Considering all of the items of comparability, Sale 2 is the most comparable to the subject. However, the results obtained from Sale 2 are well supported by Sale 1 and Sale 3. Considering all of the above factors, you select 0.050 as the overall rate without a tax component. Adding in the tax component for the tax area of the subject (0.020) provides an overall rate of 0.070. The last step is to apply the overall rate to the net operating income of the subject to produce an indication of value by direct capitalization.

Exercise 4-2

Direct Capitalization of Improved Property

You are appraising an apartment complex containing eight buildings with ten units in each building. You have conducted a market analysis and established market rent for the subject. You have also reconstructed the income/expense statement, which provided a potential gross income of $940,000, vacancy and collection loss of 5% and miscellaneous income of $7,000. Operating expenses and reserves for replacements are 30% of effective gross income. Land in this area represents 20% of a property's value. Your estimate of the remaining economic life for this property is 25 years. The assessment level is 50% of actual value and the tax rate is 40 mills.

You have found three recent sales of similar apartment complexes that do not require any adjustment for market conditions. However, the three comparables are located in different tax areas. This will require the development of an overall rate without a tax component, and then add the effective tax rate that is appropriate for the subject's location. The three sales were confirmed as arms-length transactions and sold with typical financing. A market analysis found the properties rented at market rent and you have reconstructed the income/expense statements for each comparable. Data on the three comparable sales is as follows:

	Sale 1	Sale 2	Sale 3
Sale Price	$7,200,000	$7,600,000	$8,000,000
Potential Gross Income	$936,000	$980,400	$1,108,000
Vacancy & Collection Loss	5%	5%	5%
Miscellaneous Income	$7,800	$8,220	$10,400
Expense Ratio	30%	30%	32%
Land / Improvement Ratio	1 : 4	1 : 4	1 : 3
Remaining Economic Life	23 Years	25 Years	27 Years
Effective Tax Rate	0.019	0.016	0.0175

Develop an overall rate for the three comparables that does not contain a tax component. Next, using the items of comparability, select the rate appropriate for the subject and include a tax component. What is the value of the subject?

Example 4-3

Direct Capitalization with an Overall Rate using Band of Investment Method

You are appraising a small office complex with 20,000 square feet of gross area and 1,600 square feet of common area, in an area with very limited sales. However you are able to obtain income and expense data for this type of property. You have conducted a market study and found market rent to be $15.00 per square foot of net leasable area. Typical vacancy and collection losses are 5%. Expenses are 35% of effective gross income. The effective tax rate for the subject is 0.010. Using the data obtained from your study, you have reconstructed the income/expense statement. The reconstructed statement is as follows:

Potential Gross Income	$276,000
Vacancy & Collection Loss (5%)	13,800
Effective Gross Income	262,200
Expenses (35%)	91,770
Net Operating Income	$170,430

Since there are limited sales, you decide to use the Band of Investment method to develop an overall capitalization rate (Ro) since you obtained good financial information in your market study.

Lending institutions will finance 60% of the value of the property at a rate of 8% for 25 years. Equity has been receiving a cash flow (equity dividend rate) of return of 12%. The monthly partial payment factor for an 8% loan for 25 years is 0.007718. In order to obtain the mortgage annual constant or the mortgage capitalization rate the monthly partial payment factor of .007718 is multiplied by 12 (.007718 x 12 = 0.092616).

The overall capitalization rate (without a tax component) is calculated by:

Financial Components	Percent of Investment		Rate		Product
Debt	0.60	x	0.092616	=	0.055570
Equity	0.40	x	0.12	=	0.048000
Totals	1.00				0.1035696

Overall capitalization rate without a tax component (rounded to three decimals):
0.104

Overall Capitalization Rate	0.104
Effective Tax Rate	0.010
Overall Rate with Tax Component	0.114

Indicated value of the subject property:

$170,430 / 0.114 = $1,495,000

Exercise 4-3

Direct Capitalization with an Overall Rate using Band of Investment Method

You are appraising an office complex and have obtained the following market data for the property:

Gross Square Feet	80,000
Common Area	12%
Market Rent	$18.00 per square foot of net leasable area
Vacancy Rate	10%
Expense Ratio	40%
Effective Tax Rate	0.014

Financing for this type of property can be obtained from local lenders for 70% of the value at a rate of 10%. Market analysis indicates an equity capitalization rate of 15%. The partial payment factor from the compound interest tables (column 6) for a 25-year loan at 10% with monthly payments is .009087.

Determine the indicated value of the subject property using an overall capitalization rate developed by the Band of Investment method. Round your overall capitalization rate to three decimal places.

Example 4-4

Direct Capitalization with an Overall Rate by Net Income Ratio

An office complex has a potential gross income of $270,000, with vacancy and collection loss being 10% and expenses 40% of effective gross income. The net operating income includes income to property taxes.

An analysis of similar complexes provides an effective gross income multiplier of 7.5.

The net income ratio can be obtained by subtracting the expense ratio from 1.00. The subject property has expenses of 40% or an expense ratio of 0.40. Subtracting the expense ratio from 1.00 provides a net income ratio of 0.60.

The net income ratio can also be obtained by dividing the net operating income by the effective gross income. The subject has a potential gross income of $270,000, less $27,000 for vacancy and collection loss, providing an effective gross income (EGI) of $243,000. Subtracting expenses, which are 40% of effective gross income, provides a net operating income (NOI) of $145,800. Dividing the NOI $145,800 by the EGI $243,000 provides a net income ratio of 0.60.

The overall capitalization rate is calculated as follows:
0.60 (net operating income ratio) / 7.5 (effective gross income multiplier) = 0.080 (overall capitalization rate)

Indicated value of the subject property: **$145,800 / 0.080 = $1,822,500**

Exercise 4-4

Direct Capitalization with an Overall Rate by Net Income Ratio

A retail store you are appraising has Potential Gross Income of $260,000. Vacancy and collection loss is 5%. The expense ratio is 35%. The net operating income includes income to property taxes. The effective gross income multiplier for this property is 5.

Develop an overall capitalization rate (R_o) by the Net Income Ratio method and use it to estimate the value of the subject property.

Example 4-5

Direct Capitalization with an Overall Rate by Debt Coverage Ratio

You are appraising a commercial property in an area that does not have any sales. However, you are able to obtain income and financing information. This allows you to use the Debt Coverage Ratio method to develop an overall capitalization rate (Ro).

Your subject property has a net operating income of $434,000, which includes income to property taxes.

A property comparable to your subject property is rented at market rent and has a net operating income of $450,000 with annual debt service of $360,000. The property was financed with a loan for 70% of its value, at 8%, for 20 years. The annual mortgage constant for this loan is 10%.

In order to develop an overall capitalization rate the Debt Coverage Ratio must be calculated. It is calculated as follows:

$450,000 / $360,000= 1.25

The overall rate can now be calculated by the formula:

$R_0 = DCR \times R_M \times M$

The calculation for the overall capitalization rate would be:

1.25 x 0.10 x .70 = 0.0875 or 8.75%

The indicated value for the subject property would be:

$434,000 / 0.0875 = $4,960,000

Exercise 4-5

Direct Capitalization with an Overall Rate by Debt Coverage Ratio

Your appraisal assignment is to develop an overall capitalization rate (R_o) that can be used to value a commercial property. The subject property has a net operating income of $420,000, which includes income to property taxes.

A property comparable to your subject property has a net operating income of $600,000 with annual debt service of $400,000. The property was financed with a loan for 70% of its value, at 10%, for 20 years. The annual mortgage constant for this loan is 11.5%.

Develop an overall capitalization rate (R_o) using the Debt Coverage ratio method. Use this R_O to calculate an indicated value for the subject property.

Example 4-6

Direct Capitalization with an Overall Rate by the Yield Change Formula

You are appraising a commercial retail property in a central business district. You have obtained the owners' income/expense statement and reconstructed it. The results from the reconstructed income/expense statement provide a net operating income for the subject of $400,000.

Information provided by the owner indicates an overall yield rate (Y_o) of 12%. The owner also stated that he expects the income and value to increase at a constant rate (CR) of 2% for the next five years.

Since you have obtained both the overall yield rate (Yo) and the constant rate of change (CR) and overall rate can be obtained using the formula $R_o = Y_o - CR$.

The overall capitalization rate would be: $R_o = 0.12 - 0.02$
$R_o = 0.10$ or 10%

Applying the overall capitalization rate to the subject provides an indicated value of $4,000,000.

($400,000 / 0.10= $4,000,000)

Exercise 4-6

Direct Capitalization with an Overall Rate by the Yield Change Formula

You are appraising a commercial retail property located in the suburbs. You have obtained the owners' income/expense statement and reconstructed it. The results from the reconstructed income/expense statement provide a net operating income for the subject of $320,000.

Information provided by the owner indicates an overall yield rate (Y_o) of 12%. The owner also expects the income and value to increase at a constant rate (CR) of 4% for the next five years.

Develop an overall capitalization rate (R_o) using the yield change formula and develop a value for the subject property.

Example 4-7

Gross Income (Rent) Multipliers

Example 1:
You are developing a gross income multiplier (GIM) to use with a subject you are appraising. You have found the following sales and their corresponding potential gross incomes.

Sale Number	Sale Price	Annual Gross Income	GIM
1	2,100,000	300,000	7.00
2	2,245,500	320,000	7.02
3	2,415,000	350,000	6.90
4	2,660,000	380,000	7.00

The appropriate GIM would be 7.00.

The subject you are appraising has a potential gross income of $225,000. Application of the GIM to the subject is as follows:

$225,000 x 7.0 = $1,575,000

Example 2:
You are developing an effective gross income multiplier (EGIM) to use with a subject you are appraising. You have found the following data:

Sale Number	Sale Price	Annual Gross Income	Vacancy & Collection Loss	Effective Gross Income	EGIM
1	2,400,000	420,000	20,000	400,000	6.00
2	2,250,000	410,000	30,000	380,000	5.92
3	2,675,000	465,000	25,000	440,000	6.08
4	2,880,000	510,000	30,000	480,000	6.00

The appropriate EGIM is 6.00.

The subject you are appraising has an effective gross income of $450,000. Application of the EGIM to the subject is:

$450,000 x 6.00 = $2,700,000

Exercise 4-7

Development of Gross Income (Rent) Multipliers

A retail store containing 16,000 square feet of gross leasable area is rented for $16.00 per square foot per year. Vacancy and collection loss is 4%. Allowable expenses are 40% of effective gross income. The land value is estimated to be $384,000. The store sold recently for $1,536,000.

Develop a potential gross income multiplier and an effective gross income multiplier.

Example 4-8

Description of the Compound Interest Tables (See Appendix)

Column #1 - (Future Worth of $1) - This factor is the basis for the other five columns. It shows the growth of a single deposit or investment over a specific time period. It is also known as the compound amount of $1. It is the reciprocal of Column #4 (the reversion factor).

Column #2 - (Future Worth of $1 per Period) - This factor shows the growth at compound interest of a series of deposits or investments. It is also known as the compound amount of $1 per period. It is the reciprocal of Column #3 (Sinking Fund Factor).

Column #3 - (Sinking Fund Factor) - This factor shows the periodic deposit necessary to accumulate $1 over a specific time period. It is the reciprocal of Column #2 (Future Worth of $1 per Period).

Column #4 - (Reversion Factor) - This factor shows the present worth of a single future payment or deposit of $1. It is also known as the reversion factor. It is the reciprocal of Column #1 (Future Worth of $1).

Column #5 - (Present Worth of $1 per Period) - Shows the present worth of a series of future payments or deposits. It is also known as the Inwood coefficient. It is the reciprocal of Column #6 (Periodic Repayment Factor).

Column #6 - (Partial Payment Factor) - Shows the periodic payment necessary to amortize a loan at a specified interest rate over a specific number of periods. It is also known as the Installment to Amortize $1. It is the reciprocal of Column #5 (Present Worth of $1 per Period).

Example 4-9

Use of the Compound Interest Tables (See Appendix)

Column #1 - (Future Worth of $1)

Example A single deposit of $5,000 is deposited in a savings account which pays six% annually. If the deposit is left in the account for five years, to what amount would this investment grow?

Solution $5,000 x 1.338226 = $6,691.13

Column #2 - (Future Worth of $1 per Period)

Example An annual deposit of $1,200 is deposited in a savings account which pays six% annually. If no principal or interest is withdrawn, how much will be in the savings account at the end of five years?

Solution $1,200 x 5.637093 = $6,764.51

Column #3 - (Sinking Fund Factor)

Example What annual deposit at six% interest would be required to accumulate an amount of $50,000 in 20 years?

Solution $50,000 x 0.027185 = $1,359.25

Column #4 - (Present Worth of $1)

Example On your 30th birthday you will collect the proceeds from a small insurance policy with a cash-in value of $10,000. If your 27th birthday is today, what is the present worth of your right to redeem the insurance policy assuming an annual yield of 10%?

Solution $10,000 x 0.751315 = $7,513.15

Column #5 - (Present Worth of $1 per Period)

Example A small property you own produces an annual net income of $10,000. Assuming a rate of return of six% and a remaining economic life of 12 years, what is the present worth of this level, terminal income stream?

Solution $10,000 x 8.383844 = $83,838.44

Column #6 - (Partial Payment Factor)

Example What is the annual payment on a mortgage of $550,000 at 10% interest over a 15-year term?

Solution $550,000 x 0.131474 = $72,310.70

Exercise 4-8

Application of the Compound Interest Tables (See Appendix)

You believe at the end of a lease that a property you are appraising will be worth $850,000. The lease terminates 15 years from today. What is the present value of the reversionary interest in the property today at an overall yield rate of 10%?

Exercise 4-9

Compound Interest Tables – Present Worth of an Annuity (See Appendix)

You have just won a major liability lawsuit in the amount of $12,000,000 and need advice. According to the court decision, you will be awarded an annuity in the amount of $1,000,000 per year for a 12 year period (payments to be received at the end of each year).

You also have an opportunity to purchase a home on the beach for $7,500,000 and need the money right away or you will miss out on the deal. You go to the bank to see if they will discount your court judgment annuity and pay you the present worth of your entire judgment today. The banker says they will discount your earnings at 8% and pay you the present worth today. How much is the present worth of the $12,000,000 annuity today at those terms?

Example 4-10

Valuation of Level Lease Payments with Reversion (See Appendix)

You have been asked to appraise a retail commercial property which is located in a mature neighborhood. The property is currently rented at economic rent, which produces $272,000 of net operating income annually. The lease runs for seven more years and at the end of the lease, the property is expected to be worth $4,000,000. Assuming an appropriate overall yield rate (Yo) of 10%, what is the present value of the property?

Solution:

Annual Net Operating Income	$272,000
Present Worth of $1 per period factor for 7 years	x4.868419
Present Worth of Annual Net Operating Income	**$1,324,210**
Future Value of Property in 7 Years	$4,000,000
Present Worth of $1 factor for 7 Years	x0.513158
Present Worth of Reversion	**$2,052,632**
Total Present Worth of Leased Property	**$3,376,842**

ANNUAL COMPOUND INTEREST TABLE

	Effective Rate	10.00%	Base =	1.1	Ann Convers. Periods		1
10.00 % Annual Years	1 Amount of 1 @ Comp Interest	2 Accumulation of 1 Per Period	3 Sinking Fund Factor	4 Pres Val Reversion of 1	5 Pres Val Annuity 1/Period	6 Installment to Amortize	10.00% Annual Years
1	1.100000	1.000000	1.000000	0.909091	0.909091	1.100000	1
2	1.210000	2.100000	0.476190	0.826446	1.735537	0.576190	2
3	1.331000	3.310000	0.302115	0.751315	2.486852	0.402115	3
4	1.464100	4.641000	0.215471	0.683013	3.169865	0.315471	4
5	1.610510	6.105100	0.163797	0.620921	3.790787	0.263797	5
6	1.771561	7.715610	0.129607	0.564474	4.355261	0.229607	6
7	1.948717	9.487171	0.105405	0.513158	4.868419	0.205405	7

Exercise 4-10

Compound Interest Tables – Solving for Debt Coverage Ratio (See Appendix)

You are trying to determine the debt coverage ratio for a property in order to determine an overall capitalization rate. You know the net operating income is $400,000 and the property's mortgage is an amount of $2,600,000. The financing terms call for monthly payments at a mortgage yield rate of 8% over a 15 year period. What is the debt coverage ratio?

Exercise 4-11

Compound Interest Tables – Valuation of a Level Annuity with Reversion (See Appendix)

A four-story office building is currently leased at a market rent which produces $75,000 of net operating income annually. The lease runs for 10 more years and at the end of the lease the property is expected to be worth $2,500,000. Assuming an appropriate yield rate of 6%, what is the present value of the property?

Example 4-11

Valuation of Variable Annuity with Reversion (See Appendix)

You are appraising an office building which is expected to produce net operating income as follows: $202,500 the first year, $187,500 the second year, $242,250 the third year, $257,250 the fourth year, and $270,000 the fifth year. At the end of the fifth year, the property is expected to be sold for $4,500,000. Assuming the typical investor would require a return on his investment of 12%, what is the market value of the property?

ANNUAL COMPOUND INTEREST TABLE

	Effective Rate	12.00%	Base =	1.12	Ann Convers. Periods		1
12.00%	**1**	**2**	**3**	**4**	**5**	**6**	**12.00%**
Annual Years	**Amount of 1 @ Comp Interest**	**Accumulation of 1 Per Period**	**Sinking Fund Factor**	**Pres Val Reversion of 1**	**Pres Val Annuity 1/Period**	**Installment to Amortize**	Annual **Years**
1	1.120000	1.000000	1.000000	0.892857	0.892857	1.120000	1
2	1.254400	2.120000	0.471698	0.797194	1.690051	0.591698	2
3	1.404928	3.374400	0.296349	0.711780	2.401831	0.416349	3
4	1.573519	4.779328	0.209234	0.635518	3.037349	0.329234	4
5	1.762342	6.352847	0.157410	0.567427	3.604776	0.277410	5

Year	Income	Pres. Value Factor	Present Value
1	$202,500	0.892857	$180,804
2	187,500	0.797194	149,474
3	242,250	0.711780	172,429
4	257,250	0.635518	163,487
5	4,770,000	0.567427	2,706,627
Estimated Market Value of Property			**$3,372,821**

Exercise 4-12

Valuation of Variable Annuity with Reversion (See Appendix)

An apartment complex which you are appraising is expected to produce net operating income as follows:

Year 1	$294,000
Year 2	273,000
Year 3	318,000
Year 4	308,000
Year 5	329,000

At the end of the fifth year, the property is expected to be sold for $3,000,000. From a market analysis, you have determined that the typical investor would require a return on his investment of 10% for this type of property. What is the indicated market value of the property?

Review Questions

1. _____ is a method of converting an estimate of a single year's income into value in one direct step.

2. The two generic formulas used in direct cap are:

   1. _____

   2. _____

3. _____ is a method of converting future net benefits into present value where each future net benefit is discounted at a proper yield rate (discount rate):

4. Name the four steps for performing yield capitalization:

   1. _____

   2. _____

   3. _____

   4. _____

5. A procedure used to convert periodic income and reversions into present value is called _____.

6. The _____ shows the periodic payment necessary to amortize a loan at a specified interest rate over a specific number of periods. It is also known as the installment to amortize $1.

7. The _____ shows the present worth of a series of future payments or deposits. It is also known as the Inwood coefficient.

8. The _____ shows the present worth of a single future payment or deposit of $1.

9. Write the formula for developing a gross income multiplier.

   _____

10. List three Significant comparability criteria necessary when utilizing direct capitalization.

    1. _____

    2. _____

    3. _____

11. _____ _____ _____ is the method of estimating present values of income streams by applying a present-value factor to the individual dollar amounts of cash flow expected each period.

12. A comparable sale property to the subject has a net operating income of $150,000 and sold for $1,500,000 and is located in the subject's taxing jurisdiction, which has an effective tax rate of 1%. The subject property has an expected net operating income of $132,000.

    What is the subject property's value? _____

13. A property recently sold for $2,000,000. At the time of sale the property has potential gross income of $270,000 and an expected vacancy loss of $20,000.

    What is the effective gross income multiplier? _____

14. If the anticipated sale price of your owner occupied residential property in three years is $250,000 and you want to achieve an investment rate of 10%.

    What is the value today of the property? _____

15. What is the annual payment on a mortgage of $300,000 at 10% interest over a thirty year term?

    _____

16. When a property is being appraised for ad valorem tax purposes the _____ _____ _____ must be included as a part of the overall capitalization rate.

17. List four types of amenities that must be considered when selecting vacant land sales.

    1. _____

    2. _____

    3. _____

    4. _____

18. List the four major categories of required comparability that must be considered when selecting improved property sales to use in the development of an overall capitalization rate.

    1. _____

    2. _____

    3. _____

    4. _____

19. _____ _____ is different from yield capitalization, in that it does not directly consider the individual cash flows beyond the first year.

20. The formula value equals _____ / _____ is one of the generic formulas used in direct capitalization.

Chapter 5

Objectives

Upon completion of this chapter, you will be able to:

- Understand and be able to select a proper capitalization residual technique in appraising income-producing properties.
- Find value of income-producing properties using the building residual technique.
- Find value of income-producing properties using the land residual technique.
- Analyze the shape and behavior of the income stream.
- Understand the structure of straight-line capitalization.

Introduction

This chapter is designed to introduce the students to straight-line capitalization procedures that are appropriate in certain applications. The student is taught when it is appropriate to use straight-line capitalization and the land and building residual techniques commonly used therein. In many ways straight-line capitalization provides the basis for understanding some of the other income capitalization problems and solutions encountered in income capitalization and should solidify the student's understanding of previous chapters. This chapter will enable the student to work with residual techniques and provide the basics for residual techniques used in more advanced capitalization procedures.

Historical capitalization employs the residual techniques; land residual and building residual. The residual techniques separate the property into two components, the value of one is known. Land or building residual techniques can be employed using straight-line recapture for improvements (buildings).

NOTE: Within traditional straight line capitalization, the land capitalization rate consists of the property's overall yield rate (Y_o) and the effective tax rate (ETR). Also the building (improvement) capitalization rate consists of the property's overall yield rate (Y_o), the recapture rate, and the effective tax rate. However, in direct capitalization, this is not necessarily true because the land capitalization rate (Ri_) and the land yield rate (Y_L) are not conceptually the same - just as the property's overall capitalization rate (Ro) is not necessarily

the same as the property's overall yield rate (Y_o). Likewise the building capitalization rate (R_B) and the building yield rate (Y_e) are not conceptually the same except in straight line capitalization. Thus, the various rates and relationships used in this chapter apply within the concept of historical straight line capitalization and are not necessarily appropriate for use within other capitalization methods.

Classic Straight Line Capitalization Method

A. Shape and behavior of the income stream - Straight line (downward)

B. Manner or method of handling recapture - Recapture is received in equal amounts during the economic life of the improvement (straight-line).

C. Proportion of the investment upon which the Overall Yield (discount) is earned - Non- depreciated balance of investment

D. Known value of either land or improvement

1. Land residual

2. Building residual

Straight Line Capitalization Assumptions

A. Assumes a declining income stream during the remaining economic life of the improvements

B. Recapture received in equal amounts (straight-line basis) during the remaining economic life of the improvements

C. The property's overall yield (discount) is received on the balance of the investment, after a periodic recapture of a portion of the improvement value

Components of the Land and Building Capitalization Rates

A. Land capitalization rate

1. Overall yield rate

2. Effective tax rate

B. Building capitalization rate

1. Overall yield rate

2. Effective tax rate

3. Recapture rate

a) Reciprocal of REL Method of developing the recapture rate

b) Market comparison Method of developing the recapture rate

Methods of Developing the Land and Building Capitalization Rates

A. Summation method

1. Land capitalization rate (R_L) components

a) Overall yield rate (Y_o)

b) Effective tax rate (ETR)

2. Building capitalization rate (R_e) components

a) Overall yield rate (Y_o)

b) Effective tax rate (ETR)

c) Recapture rate

B. Market comparison (using IRV formula)

1. Land income / land value = land capitalization rate (R_L)

2. Building income / building value = building capitalization

rate (R_B)

Land Residual Technique

A. A supportable improvement value must be developed for the application. The annual net return to the improvement is deducted from the total annual net operating income. The remaining income, which is the residual amount, is attributable to the land. This income is capitalized into a value indicator for the land.

B. Requirements for application:

1. Known improvement value is supported by market data.

2. Improvement must be new (or hypothetically new).

3. Improvement must represent the highest and best use of the site.

4. Develop an annual net operating income.

5. Develop an overall yield (discount) rate; recapture rate, and effective tax rate.

C. Application is appropriate when:

1. Improvements are adequate in condition and suitable to the site.

2. Land consists of a large parcel with no comparable property available

3. Land sales are not available.

4. Building is new (or hypothetical), and the proper use and cost are known.

D. Steps in the land residual technique:

1. Develop a reconstructed operating statement.
2. Determine annual net operating income.
3. Develop capitalization rates for land and improvement (building).
   a) Land capitalization rate = sum of:
      (1) Overall Yield (Discount) rate
      (2) Effective tax rate
   b) Improvement (Building) capitalization rate = sum of:
      (1) Overall Yield (Discount) rate
      (2) Recapture rate (1 / Remaining Economic Life)
      (3) Effective tax rate
4. Calculate income attributable to improvements. ($I_B = R_B \times V_B$)
5. Calculate income attributable to land.
6. Calculate indicated land value.
7. Add land and improvement values to obtain total property value.

Note: Modern capitalization theory defines the residual techniques as techniques where the investment is divided into components and allows for the capitalization of the income allocated to the component of unknown value after all investment components of known value have been satisfied. The residual technique is appropriately applied through land residual, building residual, property residual, equity residual and mortgage residual. (Property residual, equity residual and mortgage residual are not a part of this course.)

Building Residual Technique

A. The application requires the land to be valued independently of the improvement (building). The annual net return to the land is deducted from the estimated total annual net operating income. The remaining income, the residual amount, is attributable to the improvement and is capitalized into a value indicator for the building.

B. Requirements for application

1. Known land value supported by market data

2. Realistic net operating income

3. Property's Overall Yield Rate YO (discount rate), recapture rate, and effective tax rate

C. Application is appropriate when:

1. Land value estimate may be supported from computations involving a hypothetical proper improvement to the land.

2. Improvement is not new.

3. Improvement is not adequate for the site.

4. Vacant land sales are available.

D. Steps in the building residual technique

1. Develop a reconstructed operating statement.

2. Determine annual net operating income.

3. Develop capitalization rates for land and improvement (building).

   a) Land capitalization rate = sum of:

      (1) Overall Yield (Discount) rate

      (2) Effective tax rate

   b) Improvement (Building) capitalization rate = sum of:

      (1) Overall Yield (Discount) rate

      (2) Recapture rate (1 / Remaining Economic Life)

      (3) Effective tax rate

4. Calculate income attributable to land. ($I_L = R_L \times V_L$)

5. Calculate income attributable to improvement (building).

6. Calculate indicated improvement value.

7. Add land and improvement values to obtain total property value.

Example 5-1

Developing the Recapture Rate by Remaining Economic Life Method

Change years of remaining economic life to recapture rates and change recapture rates to years of remaining economic life.

Problem No.	Remaining Economic Life	Recapture Rate
1	25 Years	
2		0.050
3	50 Years	
4		0.033
5	16 Years	
6	28 Years	
7		0.022
8	60 Years	
9		0.025

Solution:

Problem No.	Remaining Economic Life	Recapture Rate
1	25 Years	***0.0400***
2	***20 Years***	0.0500
3	50 Years	***0.0200***
4	***30 Years***	0.0330
5	16 Years	***0.0625***
6	28 Years	***0.0357***
7	***45 Years***	0.0220
8	60 Years	***0.0167***
9	***40 Years***	0.0250

Example 5-2

Developing the Recapture Rate by Market Comparison Method

The net operating income produced by any property will be divided to satisfy discount, recapture, and real estate taxes. When any two of these are known, the third can readily be obtained.

A recapture rate can be developed from comparable sale properties when the following information is known:

Sale price
Land or improvement value
Net operating income
Overall yield rate (discount rate)
Effective tax rate

Given the following data:

Sale Price	$1,600,000
Land Value	400,000
Net Operating Income	198,000
Overall Yield Rate (Y_o)	0.085
Effective Tax Rate	0.020

Derive the recapture rate using the market comparison method.

Calculation of Net Operating Income After Deduction of Discount and Taxes

Net Operating Income	$ 198,000
Discount Dollars ($1,600,000 x 0.085)	-136,000
Tax Dollars ($ 1,600,000 x 0.020)	-32,000
Net Operating Income After Deduction for Discount and Taxes	$ 30,000

Calculation of Recapture Rate

Recapture Income	$ 30,000
Sale Price (improvements)	$1,200,000

Recapture Rate ($30,000 / $1,200,000)	0.025

Exercise 5-1

Developing a Recapture Rate by Market Comparison Method

Given the following data:

Sale Price	$1,800,000
Improvement Value	$1,200,000
Net Operating Income	$240,000
Overall yield (discount) rate	8.5%
Assessment Level	25%
Tax Rate	80 mills

Calculate the recapture rate using the market comparison method.

Example 5-3

Developing Land Capitalization Rates by Market Comparison & Summation Methods

Developing a land capitalization rate from market sales transactions requires the appraiser to identify the land income and the land value, which can be used with the IRV equation to determine the land capitalization rate. Developing a land capitalization rate by the summation method requires the appraiser to sum the overall property yield rate and the effective tax rate. Both methods are demonstrated below.

Given the following:

Sale Price	$2,000,000
Improvement Value	1,600,000
Land Income	40,000
Assessment Level	50%
Tax Rate	$4.00 per $100
Overall Property Yield Rate	8%

Market Comparison Method:

Land income / land value = land rate = $40,000 / $400,000 = 10%

(Land Value = Sale Price of $2,000,000 Less Improvement Value of

$1,600,000)

or

Summation Method:

Overall Property Yield (Discount) Rate	0.08
Effective Tax Rate:	
(Assessment Level) 0.50 x 0.04 (Tax Rate)	0.02
Land Capitalization Rate	0.10

Exercise 5-2

Developing a Land Capitalization Rate by Market Comparison & Summation Methods

Given the following:

Land Value	$500,000
Assessment Level	50%
Tax Rate	40 mills
Discount or Overall Yield Rate (Y_o)	8.5%
Remaining economic Life	25 Years
Land Income	$52,500

A. Determine the land capitalization rate (R_L) by summing the overall property yield rate and effective tax rate.

B. Determine the land capitalization rate (R_L) by using the IRV equation.

Example 5-4

Developing a Building Capitalization Rates by Market Comparison & Summation Methods

Developing an improvement (building) capitalization rate from market sales transactions requires the appraiser to identify the improvement income and the improvement value, which can be used with the IRV equation to determine the improvement capitalization rate. Developing an improvement capitalization rate by the summation method requires the appraiser to sum the Y_o (discount rate), recapture rate, and the effective tax rate. Both methods are demonstrated below.

Sale Price	$2,000,000
Land Value	400,000
Improvement Income	240,000
Assessment Level	50%
Tax Rate	
$4.00 per $100	
Remaining Economic Life	20 years
Overall Yield Rate (Y_0)	8%

Market Comparison Method:

Improvement income / improvement value =

improvement rate $240,000 / $1,600,000 = 0.15 or

15%

Summation Method:

Y_o (Discount Rate:)	0.08
Effective Tax Rate:	
(Assessment Level) 0.50 x 0.04 (Tax Rate) =	0.02
Recapture Rate:	
1 / 20 (Remaining Economic Life) =	0.05
Improvement Capitalization Rate =	0.15 or 15%

Exercise 5-3

Developing a Building Capitalization Rate by Market Comparison & Summation Methods

Given the following:

Sale Price	$1,125,000
Land Value	$ 225,000
Assessed Value	$ 281,250
Tax Rate	80 mills
Overall Yield Rate (Discount Rate)	10%
Remaining Economic Life	25 years
Net Operating Income	$ 171,000

A. Determine the improvement capitalization rate by summing the overall yield rate, recapture rate and effective tax rate.

B. Determine the improvement capitalization rate by using the IRV equation.

Example 5-5

Straight Line Capitalization – Land Residual Technique

Your assignment is to appraise a parcel of vacant land, in the middle of a business block in a desirable community. Zoning is for commercial purposes, and building codes allow full coverage of the site. The land has 100 front feet on the street and a uniform depth of 150 feet for rectangular shape.

A study of the highest and best use of the site indicated that a one-story brick structure housing two retail stores would be most profitable. The proposed building would cover the entire site, using all the space. A contractor reports the cost of the proposed structure is $125.00 per square foot. Net operating income is estimated to be $368,750 for the proposed improved property.

Analysis indicates the following rates are appropriate: overall yield rate, 12%; recapture, 2%; and effective tax rate, 2%. Further analysis indicates that straight line capitalization should be employed with this proposed improved property.

Because of the absence of comparable land sales, the land residual technique is used to arrive at an estimate of value for this parcel.

Application of Land Residual Technique

Net Operating Income	$368,750
Less Income Attributable to Building	
(15,000 s.f. @ $125.00 = $1,875,000 x 16%)	- 300,000
Residual Income to Land	$68,750

Land Capitalization Rate (R_L):

Property Discount Rate (Y_o) or overall yield on investment	0.12
Effective Tax Rate	0.02
Land Capitalization Rate	0.14

Residual Income to Land (Capitalized at 0.14)

$491,071

Rounded to

$491,000

Example 5-6

Straight Line Capitalization – Template

Net Operating Income			$ _____
Less: **Income to land (or improvement)**			
(Land value x land rate)			
(or Improvement value x improvement rate)			– _____
Residual income to improvement (or land)			
Improvement income capitalized at improvement cap rate			$ _____
(Land capitalized at land cap rate)			
Property yield rate (discount rate)	_____		
Effective tax rate	_____		
Total land capitalization rate	=	_____	
Recapture rate (return of improvement investment)		_____	
Total improvement capitalization rate	=	_____	
$\frac{\text{Improvement income}}{\text{Improvement cap rate}}$ = Improvement value			$ _____
Or			
$\frac{\text{Land income}}{\text{Land cap rate}}$ = Land value			$ _____
Total property value			$ _____

Example 5-7
Straight Line Capitalization – Matrix

	Income	Rate	Value
Building			
Land			
Total			

Exercise 5-4

Straight Line Capitalization – Land Residual Technique

You are appraising a downtown parking lot that has recently been paved with asphalt over a crushed stone base at a cost $3.00 per square foot. The paving has an estimated life expectancy of 10 years. The lot has a frontage of 250 feet and a depth of 400 feet.

Annual anticipated operating expenses are listed below.

Insurance $2,400
Management ..... 4% of effective gross income
Lot maintenance $18,000
Legal and accounting ..... 1% of effective gross income
Miscellaneous expenses $3,600
Real estate taxes ..... 2.5% of the value
Typical investor expected rate of return.....9.5%
Effective gross income $225,000

Using straight-line capitalization, estimate the value of the property.

Exercise 5-5

Straight Line Capitalization – Land Residual Technique

The appraisal assignment is a parcel of vacant land located in an area of improved strip commercial property. Zoning is for commercial use, and building codes allow for full improvement development on the site. The lot is rectangular in shape, with a frontage of 100 feet and a depth of 125 feet.

A highest and best use analysis indicates the most profitable use of the site is retail, with a one-story building utilizing the entire site.

Contractor estimates for cost of construction of the proposed building is $140.00 per square foot, with an economic life of 40 years.

Area market rent and expense analysis produced the following:

Market rent	$35 per square foot annually
Vacancy and collection loss	2%
Management fees	5%
Insurance	$7,700
Maintenance and repair	$ 15,750
Utilities	$12,600
Trash service	$2,100
Legal and accounting	$4,200
Miscellaneous expense	$5,600
Replacement reserves	$7,000

Further analysis produced the following:

Overall yield rate for property (Y_o)	12%
Assessment level	25%
Tax rate	60 mills

Using the above information, develop a value estimate for the land.

Example 5-8

Straight Line Capitalization – Land Residual Technique

The subject property of this appraisal is a supermarket containing 12,000 square feet, with a land value of $800,000 derived from recent sales. The property is zoned commercial and is currently leased at market rent to a national food store chain on a 10-year lease at $30 per square foot per year. A vacancy and collection loss allowance of 5% is indicated to reflect the credit rating of the tenant and the term of the lease.

Allowable expenses consist of:

Insurance - $4,280
Maintenance and repairs - $7,200
Management of property- 5% of effective gross income
Legal and accounting fees - 1% of effective gross income
Parking lot maintenance - $4,800 per year

Real estate taxes amount to 2.6% of the value of the property. The land should return 6% yield plus the 2.6% for taxes.

The capitalization rate for the building should include 6% overall yield, 2% for recapture (50 year estimated remaining life of the building), plus the 2.6% for taxes.

Given this data, you have reconstructed the income and expense statement as shown below:

Potential Gross Income (12,000 SF @ $30.00)	$360,000
Less Vacancy and Collection Loss (5%)	- 18,000
Effective Gross Income	$342,000
Less Expenses:	
Management ($342,000 x 5%)	$17,100
Legal & Accounting ($342,000 x 1%)	3,420
Parking Lot Maintenance	4,800
Maintenance and Repair	7,200
Insurance	4,280
Total Expenses	- 36,800
Net Operating Income	$305,200

The next step is to value the property by the straight-line capitalization method, building residual technique as shown on the following page.

Example 5-8

Straight Line Capitalization – Land Residual Technique – (continued)

Net Operating Income		$305,200
Less income attributable to the land ($800,000 x 0.086)=		- 68,800
Residual income attributable to the building		236,400
Capitalized at 0.106		$2,230,189
Overall yield rate	0.060	
Recapture of building portion of investment	0.020	
Effective Tax Rate	0.026	
Building Capitalization Rate	0.106	
Plus land value		+ 800,000
		3,030,189
	Rounded to	**$3,030,000**

Straight Line Capitalization Matrix

	Income	Rate	Value
Building			
Land			
Total			

Exercise 5-6

Straight Line Capitalization – Building Residual Technique

The Orange Grove office building, containing 20 equal size offices of 1,200 square feet each, was constructed 10 years ago. After reviewing today's market, you have estimated the remaining economic life of the building to be 25 years. The current market rent for this type of office space is $25 per square foot. The typical operating expenses for this property amount to $171,000 annually. Vacancy and collection loss is expected to be 5%, similar to other office buildings in the area.

After reviewing the real estate market, you find the typical overall property yield rate for similar investments is 9% and real estate taxes are 2.5%. The land, on which the Orange Grove office building sits, has recently been valued at $600,000 using comparable sales.

Using straight-line capitalization and the building residual technique, compute the indicated value of the improved property.

Exercise 5-7

Straight Line Capitalization – Building Residual Technique

You are appraising an older industrial building, which has a remaining economic life of 20 years, in an urban industrial district. The land where the building is situated is valued by comparable sales at $2,500,000. The potential gross income for the property is expected to be $2,354,000 and the vacancy and collection loss for this type of building is typically 4%. Operating expenses are expected to be $660,000.

Typical investors require an overall yield rate of 8% to invest in this type of property and property taxes are based on a $4.00/per $100 tax rate with an assessment level of 50%.

This property fits the requirements for straight line capitalization. Your assignment is to appraise the property using straight line capitalization with the building residual technique.

Review Questions

1. If the land capitalization rate in straight line capitalization (including the effective tax rate) is 9% and the remaining economic life of the improvement is 25 years, the building capitalization rate is _____.

2. In straight line capitalization, _____ is received in equal amounts during the remaining economic life of the improvement.

3. Straight line capitalization assumes a _____ income stream during the remaining economic life of the improvements.

4. Straight line capitalization is best utilized when the property has:

   1. _____

   2. _____

   3. _____

5. The building residual technique should be used when:

   1. _____

   2. _____

   3. _____

   4. _____

Review Questions – (continued)

6. The land residual technique should be used when:

   1. _____
   2. _____
   3. _____
   4. _____

7. The land capitalization rate is composed of what components?

   1. _____
   2. _____

8. The building capitalization rate is composed of what components?

   1. _____
   2. _____
   3. _____

9. The overall yield rate (Y_o) is another name for the property's _____ rate.

10. To obtain the land income in the building residual method one _____ the land capitalization rate by the _____ value.

Review Questions – (continued)

11. To obtain the _____ capitalization rate, one can divide the building income by the building _____.

12. The _____ residual technique requires the development of the discount rate, recapture rate, effective tax rate, and a supportable market value for land.

    A. Land
    B. Building
    C. Both A & B
    D. None of the above

Quiz #2

1. A gross income multiplier (GIM), as used in a commercial appraisal, is obtained by dividing the:

   A. sale price by annual gross income
   B. sale price by monthly gross income
   C. overall capitalization rate by the sale price
   D. annual gross income by the sale price

2. The borrowing of funds in hopes of earning a greater return than the cost of the borrowed funds is known as:

   A. appreciation
   B. liquidity
   C. leverage
   D. none of the above

3. Direct capitalization is a valid method when the overall rate is developed from sales in which:

   A. the land-to-building ratios are similar to those of the subject property
   B. the remaining economic lives are similar to those of the subject property
   C. the income and expense ratios are similar to those of the subject property
   D. all of the above

4. The subject property's net income is $15,000 per year. Comparable investments sold and are reported below.

Comparable	Net Income	Sales Price
1	$14,400	$120,000
2	$14,000	$147,400
3	$13,500	$122,700
4	$14,500	$152,600

All of the comparables recently sold; comparables 2 and 4 are most similar to the subject property. Using direct capitalization with an overall rate, what is the best estimate of the value of the subject property (rounded to the nearest $1,000)?

A. $125,000
B. $137,000
C. $143,000
D. $158,000

5. One of the standard techniques used by appraisers when analyzing the income of property is to divide the income being generated between the land and the improvements and then to capitalize the residual income to the land into a value estimate. This technique is referred to as the:

A. land residual technique
B. building residual technique
C. property residual technique
D. plottage residual technique

6. The direct relationship between a single year's annual net operating income and the property's sale price value is:

A. direct capitalization
B. immediate capitalization
C. overall capitalization rate
D. future capitalization

7. A property has a land value of $1,000,000, a net operating income of $350,000, a land capitalization rate of 10 percent, and a building capitalization rate of 12 1/2 percent. What is the value of the subject property?

   A. $1,500,000
   B. $2,000,000
   C. $2,500,000
   D. $3,000,000

8. A property comparable to your subject property has a net operating income of $1,200,000 with an annual debt service of $800,000. The property was financed with a loan for 70% of its value, at 7.95% for 20 years. The annual mortgage constant for this loan is 10%. Your subject property has a net operating income of $840,000. What is the indicated subject value?

   A. $7,850,000
   B. $8,000,000
   C. $8,400,000
   D. $7,600,000

9. A property has annual net operating income of $121,000. Annual principal paid on the mortgage is $10,500, annual interest payments total $94,500. What is the debt coverage ratio?

   A. 2.15
   B. 2.10
   C. 1.20
   D. 1.15

10. A comparable property's 'rents receivable' amounted to $480,000 but only collected $470,400. What is the collection loss?

   A. 0.020
   B. 0.025
   C. 0.030
   D. 0.035

11. Using the Band of Investment technique, develop the overall capitalization rate from the following information and appraise your subject property, which has a net operating income of $1,040,928. The effective tax rate is 1%.

    Mortgage = 80% of value, at 8% monthly, with a 25 year term
    Equity capitalization rate = 10%

    A. $11,000,000
    B. $10,000,000
    C. $12,000,000
    D. $13,000,000

12. You are appraising a commercial property by the income approach. You have net operating income of $400,000. You estimate the overall yield rate (discount rate) to be 10%, the recapture rate to be 4% and the effective tax rate to be 1%. The land value is $800,000. What is the indicated value of the property by the income approach?

    A. $2,080,000
    B. $2,880,000
    C. $3,120,000
    D. $3,880,000

13. In straight-line capitalization _____ is received in equal amounts during the economic life of the improvement.

    A. rent
    B. recapture
    C. principal
    D. property tax

14. Given the following data, determine the improvement capitalization rate by using the IRV equation.

Sale Price	$1,125,000
Land Value	$ 225,000
Assessed Value	$ 281,250
Tax Rate	80 mills
Overall Yield Rate (Discount Rate)	10%
Remaining Economic Life	25 years
Net Operating Income	$ 171,000

A. 0.14
B. 0.15
C. 0.16
D. 0.17

15. A retail store containing 32,000 square feet of gross leasable area is rented for $18.00 per square foot per year. Vacancy and collection loss is 4 percent. Allowable expenses are 40 percent of effective gross income. The land value is estimated to be $ 384,000. The store sold recently for $3,317,760. Develop an effective gross income multiplier from this data.

A. 5.75
B. 7.00
C. 8.50
D. 6.00

Appendix

Appendix -- Rates and Relationships

IRV

$I = R \times V$
$R = I \div V$
$V = I \div R$

VIF

$V = I \times F$
$I = V \div F$
$F = V \div I$

EAT

$E = A \times T$
$A = E \div T$
$T = E \div A$

$R_O = Y_O - CR$
$Y_O = R_O + CR$
$CR = Y_O - R_O$

$DCR = NOI \div I_M$

$R_O = DCR \times M \times R_M$

$R_O = NIR \div EGIM$

$R_O = M \times R_M + (1 - M) \times R_E$

$R_L = I_L \div V_L$

$R_B = I_B \div V_B$

$R_E = I_E \div V_E$

$R_M = I_M \div V_M$

$NIR = 1 - OER$

$OER = 1 - NIR$

GIM = Property Value ÷ Gross Income

EGIM = Property Value ÷ Effective Gross Income

Description of Various Rates

Overall Capitalization Rate (R_O) – The R_O reflects the relationship between a single years's net operating income and the total property value. It is typically derived from market comparisons with highly comparable sales, band of investment calculations, the DCR technique, NIR ÷ GIM, yield change techniques. It also reflects the weighted average of the R_L and R_B weighted by the respective contributions to value of the land and building. R_O can be either greater than, equal to, or less than Y_O depending on whether the property value is expected to decrease, remain stable, or increase respectively.

Overall Yield Rate (Y_O) – Sometimes referred to as the property discount rate or "r" or the "return on" rate, internal rate of return, or property interest rate. The overall yield rate is expressed as a compound annual percentage rate. It considers all expected benefits of property ownership, including the proceeds from the sale at the termination of the investment. It can be obtained by extraction from sales of similar properties, comparison with alternative investments of comparable risk, surveys of property investors, or the yield change formula – $Y_O = R_O + CR$. [CAUTION: Using the band of investment technique to blend Y_M and Y_E is mathematically incorrect unless the loan is "interest only" and there is no change in income or value over the holding period. Conceptually, the Y_O is a weighted average of the equity yield rate and the mortgage interest rate. However, in practice the Y_O is not usually calculated by the BOI technique because the ratio of debt and equity changes each year as the loan is amortized and the property value changes.]

Land Capitalization Rate (R_L) – The R_L reflects the relationship between a single year's net income attributable to the land and the value of the land. It is usually extracted from sales using the relationship $R_L = I_L \div V_L$. R_L can be either greater than, equal to, or less than Y_L depending on whether the land value is expected to decrease, remain stable, or increase respectively. [Note: Within the context of classic straight line capitalization where land income and value are not expected to change over the holding period, the land capitalization rate (including a property tax component) is the sum of the Y_O and the effective tax rate (because in that situation $R_L = Y_L$).]

Land Yield Rate (Y_L) – The Y_L reflects the rate of return on capital invested in the land portion of real property. It is expressed as a compound annual rate and considers all expected benefits of land ownership, including the proceeds from the sale of the land at the termination of the investment. Since $Y_L = Y_O = Y_B$ is virtually always true (because investors rarely assign a different risk rate to the land and building), the same methods used to derive Y_O are valid for deriving Y_L.

Building (Improvement) Capitalization Rate (R_B) – The R_B reflects the relationship between a single year's net income attributable to the building and the value of the building. It is usually extracted from sales using the relationship $R_B = I_B \div V_B$. R_B can be either greater than, equal to, or less than Y_B depending on whether the building value is expected to decrease, remain stable, or increase respectively. [Note: Within the context of classic straight line capitalization where building "return on" income is earned only on the undepreciated balance of the building investment during the holding period, the building capitalization rate (including a property tax component) is the sum of the Y_O, the recapture rate, and the effective tax rate (because in that situation, $R_B = Y_B$).]

Building (Improvement) Yield Rate (Y_B) – The Y_B reflects the rate of return on capital invested in the building portion of real property. It is expressed as a compound annual rate and considers all expected benefits of building ownership, including the proceeds from the sale of the building at the termination of the investment. Since $Y_B = Y_O = Y_L$ is virtually always true (because investors rarely assign a different risk rate to the land and building), the same methods used to derive Y_O are valid for deriving Y_B.

Mortgage Capitalization Rate (R_M) – Also known as the mortgage annual constant, the R_M is the ratio of the annual debt service (principal & interest). It reflects the relationship between mortgage income (I_M) and mortgage principal. It is one of the components used in the band of investment technique to get an R_O. It can be obtained from the compound interest tables by multiplying the partial payment factor (Column #6) by the number of conversion periods in one year.

Mortgage Yield Rate (Y_M) – The Y_M is also known as the mortgage interest rate. It is the lender's rate of return on a loan (if no points or other charges increase the lender's yield). It is usually obtained by interviewing lenders or can be extracted from the terms of the mortgage.

Equity Capitalization Rate (R_E) – The R_E is also known as the equity dividend rate or the cash on cash rate. It reflects the relationship between a single year's equity income and the value of the equity. It is typically extracted from sales using the relationship $R_E = I_E \div V_E$. It is extremely sensitive to loan-to-value ratios. [CAUTION: Unlike the Y_E, the R_E should not be derived from sales of stocks or bonds because of dissimilarity of investments, i.e. liquidity, holding term, etc.]

Equity Yield Rate (Y_E) – The Y_E is the "return on" rate for equity and is sometimes referred to as the equity discount rate or the equity interest rate. It considers all the expected cash flows attributable to the equity investment, including proceeds from sale at the termination of the investment. The Y_E can be extracted from sales of similar properties, from alternative investments of comparable risk such as stocks and bonds, or from surveys of market participants.

Loan to Value Ratio (M) – This is the ratio between a mortgage loan and the value of the property, usually expressed as a percentage. It is usually obtained from interviews with active lenders. It can be extracted from sales by dividing the loan amount by the selling price.

Debt Coverage Ratio (DCR) – This is the ratio of net operating income to annual debt service (principal & interest). It measures the ability of a property to meet its annual debt requirements out of net operating income. The DCR can be extracted from sales by dividing the net operating income by the annual debt service. The DCR is an ideal test of reasonableness and can be multiplied by the R_M and the M to obtain R_O. ($R_O = DCR \times R_M \times M$)

Effective Tax Rate (ETR) – This is the ratio of property taxes to total property value. It can be derived by the EAT formula by multiplying the assessment level times the tax rate as long as all components are expressed as proper decimals. ($ETR = A \times T$) It can also be extracted from sales by dividing the income to property taxes by total property value. It can be directly added to or subtracted from the R_O to get an R_O with or without a property tax component. Conceptually, the ETR cannot be directly added to or subtracted from the Y_O to get a Y_O with or without a property tax component because Y_O is a yield rate used in discounting the cash flows in yield capitalization and the ETR is a capitalization rate. An exception to this is when using classic straight line capitalization where the $Y_O = R_L$.

Operating Expense Ratio (OER) – The OER reflects the relationship between operating expenses and the Effective Gross Income (EGI) or between the relationship between operating expenses and Potential Gross Income. Consistency is paramount to using this ratio properly. If the OER is represented to be a percentage of EGI it must be applied consistently with the manner in which it was derived. Likewise, if the OER is represented to be a percentage of Potential Gross Income (PGI) it must be applied consistently with the manner in which it was derived.

Net Income Ratio (NIR) – The NIR reflects the relationship between net operating income and the Effective Gross Income (EGI) or between the relationship between net operating income and Potential Gross Income (PGI). Consistency is paramount to using this ratio properly. If the NIR is represented to be a percentage of EGI it must be applied consistently with the manner in which it was derived. Likewise, if the NIR is represented to be a percentage of (PGI) it must be applied consistently with the manner in which it was derived.

Abbreviations Used in Income Approach to Value

BOI	Band of Investment
CR	Constant Rate of Change
DCR	Debt Coverage Ratio
EAT	Effective Tax Rate, Assessment Level, & Tax Rate formula
EGI	Effective Gross Income
EGIM	Effective Gross Income Multiplier
ETR	Effective Tax Rate
GIM	Gross Income Multiplier (sometimes known as Gross Rent Multiplier)
GLA	Gross Leasable Area
GRM	Gross Rent Multiplier (sometimes known as Gross Income Multiplier)
I_B	Income to the Building (Improvement)
I_E	Income to the Equity
I_L	Income to the Land
I_M	Income to the Mortgage (income necessary to pay principal & interest)
IRV	Income Rate & Value formula
I_V	Income to total property value (Net Operating Income)
NIR	Net Income Ratio (also known as Net Operating Income Ratio or 1 - OER)
NLA	Net Leasable Area
NOI	Net Operating Income
OER	Operating Expense Ratio (or 1 - NIR)
PAV	Property Assessment Valuation (published 1996)
PGI	Potential Gross Income
PW	Present Worth
R_B	Building Capitalization Rate (also known as the Improvement Capitalization Rate)
R_E	Equity Capitalization Rate (also known as the Equity Dividend Rate or Cash on Cash Rate)
R_L	Land Capitalization Rate
R_M	Mortgage Capitalization Rate
R_O	Overall Capitalization Rate (also known as OAR)
V_B	Value of the Building (Improvement)
V_E	Value of Equity
VIF	Value Income & Factor formula
V_L	Value of the Land
V_M	Value of the Mortgage (Mortgage Principal)
V&C	Vacancy & Collection Loss
Y_B	Building Yield Rate
Y_E	Equity Yield Rate
Y_L	Land Yield Rate
Y_M	Mortgage Yield Rate (Mortgage Interest Rate or lender's yield on the loan)
Y_O	Overall Yield Rate (also known as the Discount Rate or "r")

Compound Interest Tables

Compound Interest Tables

Annual Compound Interest tables follow:

6% Annual Compound Interest Table

Period	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.060000	1.000000	1.000000	0.943396	0.943396	1.06000000
2	1.123600	2.060000	0.485437	0.889996	1.833393	0.54543689
3	1.191016	3.183600	0.314110	0.839619	2.673012	0.37410981
4	1.262477	4.374616	0.228591	0.792094	3.465106	0.28859149
5	1.338226	5.637093	0.177396	0.747258	4.212364	0.23739640
6	1.418519	6.975319	0.143363	0.704961	4.917324	0.20336263
7	1.503630	8.393838	0.119135	0.665057	5.582381	0.17913502
8	1.593848	9.897468	0.101036	0.627412	6.209794	0.16103594
9	1.689479	11.491316	0.087022	0.591898	6.801692	0.14702224
10	1.790848	13.180795	0.075868	0.558395	7.360087	0.13586796
11	1.898299	14.971643	0.066793	0.526788	7.886875	0.12679294
12	2.012196	16.869941	0.059277	0.496969	8.383844	0.11927703
13	2.132928	18.882138	0.052960	0.468839	8.852683	0.11296011
14	2.260904	21.015066	0.047585	0.442301	9.294984	0.10758491
15	2.396558	23.275970	0.042963	0.417265	9.712249	0.10296276
16	2.540352	25.672528	0.038952	0.393646	10.105895	0.09895214
17	2.692773	28.212880	0.035445	0.371364	10.477260	0.09544480
18	2.854339	30.905653	0.032357	0.350344	10.827603	0.09235654
19	3.025600	33.759992	0.029621	0.330513	11.158116	0.08962086
20	3.207135	36.785591	0.027185	0.311805	11.469921	0.08718456
21	3.399564	39.992727	0.025005	0.294155	11.764077	0.08500455
22	3.603537	43.392290	0.023046	0.277505	12.041582	0.08304557
23	3.819750	46.995828	0.021278	0.261797	12.303379	0.08127848
24	4.048935	50.815577	0.019679	0.246979	12.550358	0.07967900
25	4.291871	54.864512	0.018227	0.232999	12.783356	0.07822672
26	4.549383	59.156383	0.016904	0.219810	13.003166	0.07690435
27	4.822346	63.705766	0.015697	0.207368	13.210534	0.07569717
28	5.111687	68.528112	0.014593	0.195630	13.406164	0.07459255
29	5.418388	73.639798	0.013580	0.184557	13.590721	0.07357961
30	5.743491	79.058186	0.012649	0.174110	13.764831	0.07264891
31	6.088101	84.801677	0.011792	0.164255	13.929086	0.07179222
32	6.453387	90.889778	0.011002	0.154957	14.084043	0.07100234
33	6.840590	97.343165	0.010273	0.146186	14.230230	0.07027293
34	7.251025	104.183755	0.009598	0.137912	14.368141	0.06959843
35	7.686087	111.434780	0.008974	0.130105	14.498246	0.06897386
36	8.147252	119.120867	0.008395	0.122741	14.620987	0.06839483
37	8.636087	127.268119	0.007857	0.115793	14.736780	0.06785743
38	9.154252	135.904206	0.007358	0.109239	14.846019	0.06735812
39	9.703507	145.058458	0.006894	0.103056	14.949075	0.06689377
40	10.285718	154.761966	0.006462	0.097222	15.046297	0.06646154

6% Monthly Compound Interest Table

Month	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.005000	1.000000	1.000000	0.995025	0.995025	1.00500000
2	1.010025	2.005000	0.498753	0.990075	1.985099	0.50375312
3	1.015075	3.015025	0.331672	0.985149	2.970248	0.33667221
4	1.020151	4.030100	0.248133	0.980248	3.950496	0.25313279
5	1.025251	5.050251	0.198010	0.975371	4.925866	0.20300997
6	1.030378	6.075502	0.164595	0.970518	5.896384	0.16959546
7	1.035529	7.105879	0.140729	0.965690	6.862074	0.14572854
8	1.040707	8.141409	0.122829	0.960885	7.822959	0.12782886
9	1.045911	9.182116	0.108907	0.956105	8.779064	0.11390736
10	1.051140	10.228026	0.097771	0.951348	9.730412	0.10277057
11	1.056396	11.279167	0.088659	0.946615	10.677027	0.09365903
12	1.061678	12.335562	0.081066	0.941905	11.618932	0.08606643
Year						
1	1.061678	12.335562	0.081066	0.941905	11.618932	0.08606643
2	1.127160	25.431955	0.039321	0.887186	22.562866	0.04432061
3	1.196681	39.336105	0.025422	0.835645	32.871016	0.03042194
4	1.270489	54.097832	0.018485	0.787098	42.580318	0.02348503
5	1.348850	69.770031	0.014333	0.741372	51.725561	0.01933280
6	1.432044	86.408856	0.011573	0.698302	60.339514	0.01657289
7	1.520370	104.073927	0.009609	0.657735	68.453042	0.01460855
8	1.614143	122.828542	0.008141	0.619524	76.095218	0.01314143
9	1.713699	142.739900	0.007006	0.583533	83.293424	0.01200575
10	1.819397	163.879347	0.006102	0.549633	90.073453	0.01110205
11	1.931613	186.322629	0.005367	0.517702	96.459599	0.01036703
12	2.050751	210.150163	0.004759	0.487626	102.474743	0.00975850
13	2.177237	235.447328	0.004247	0.459298	108.140440	0.00924723
14	2.311524	262.304766	0.003812	0.432615	113.476990	0.00881236
15	2.454094	290.818712	0.003439	0.407482	118.503515	0.00843857
16	2.605457	321.091337	0.003114	0.383810	123.238025	0.00811438
17	2.766156	353.231110	0.002831	0.361513	127.697486	0.00783101
18	2.936766	387.353194	0.002582	0.340511	131.897876	0.00758162
19	3.117899	423.579854	0.002361	0.320729	135.854246	0.00736083
20	3.310204	462.040895	0.002164	0.302096	139.580772	0.00716431
21	3.514371	502.874129	0.001989	0.284546	143.090806	0.00698857
22	3.731129	546.225867	0.001831	0.268015	146.396927	0.00683074
23	3.961257	592.251446	0.001688	0.252445	149.510979	0.00668847
24	4.205579	641.115782	0.001560	0.237779	152.444121	0.00655978
25	4.464970	692.993962	0.001443	0.223966	155.206864	0.00644301
30	6.022575	1004.515042	0.000996	0.166042	166.791614	0.00599551
35	8.123551	1424.710299	0.000702	0.123099	175.380226	0.00570190
40	10.957454	1991.490734	0.000502	0.091262	181.747584	0.00550214

7% Annual Compound Interest Table

Period	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.070000	1.000000	1.000000	0.934579	0.934579	1.07000000
2	1.144900	2.070000	0.483092	0.873439	1.808018	0.55309179
3	1.225043	3.214900	0.311052	0.816298	2.624316	0.38105167
4	1.310796	4.439943	0.225228	0.762895	3.387211	0.29522812
5	1.402552	5.750739	0.173891	0.712986	4.100197	0.24389069
6	1.500730	7.153291	0.139796	0.666342	4.766540	0.20979580
7	1.605781	8.654021	0.115553	0.622750	5.389289	0.18555322
8	1.718186	10.259803	0.097468	0.582009	5.971299	0.16746776
9	1.838459	11.977989	0.083486	0.543934	6.515232	0.15348647
10	1.967151	13.816448	0.072378	0.508349	7.023582	0.14237750
11	2.104852	15.783599	0.063357	0.475093	7.498674	0.13335690
12	2.252192	17.888451	0.055902	0.444012	7.942686	0.12590199
13	2.409845	20.140643	0.049651	0.414964	8.357651	0.11965085
14	2.578534	22.550488	0.044345	0.387817	8.745468	0.11434494
15	2.759032	25.129022	0.039795	0.362446	9.107914	0.10979462
16	2.952164	27.888054	0.035858	0.338735	9.446649	0.10585765
17	3.158815	30.840217	0.032425	0.316574	9.763223	0.10242519
18	3.379932	33.999033	0.029413	0.295864	10.059087	0.09941260
19	3.616528	37.378965	0.026753	0.276508	10.335595	0.09675301
20	3.869684	40.995492	0.024393	0.258419	10.594014	0.09439293
21	4.140562	44.865177	0.022289	0.241513	10.835527	0.09228900
22	4.430402	49.005739	0.020406	0.225713	11.061240	0.09040577
23	4.740530	53.436141	0.018714	0.210947	11.272187	0.08871393
24	5.072367	58.176671	0.017189	0.197147	11.469334	0.08718902
25	5.427433	63.249038	0.015811	0.184249	11.653583	0.08581052
26	5.807353	68.676470	0.014561	0.172195	11.825779	0.08456103
27	6.213868	74.483823	0.013426	0.160930	11.986709	0.08342573
28	6.648838	80.697691	0.012392	0.150402	12.137111	0.08239193
29	7.114257	87.346529	0.011449	0.140563	12.277674	0.08144865
30	7.612255	94.460786	0.010586	0.131367	12.409041	0.08058640
31	8.145113	102.073041	0.009797	0.122773	12.531814	0.07979691
32	8.715271	110.218154	0.009073	0.114741	12.646555	0.07907292
33	9.325340	118.933425	0.008408	0.107235	12.753790	0.07840807
34	9.978114	128.258765	0.007797	0.100219	12.854009	0.07779674
35	10.676581	138.236878	0.007234	0.093663	12.947672	0.07723396
36	11.423942	148.913460	0.006715	0.087535	13.035208	0.07671531
37	12.223618	160.337402	0.006237	0.081809	13.117017	0.07623685
38	13.079271	172.561020	0.005795	0.076457	13.193473	0.07579505
39	13.994820	185.640292	0.005387	0.071455	13.264928	0.07538676
40	14.974458	199.635112	0.005009	0.066780	13.331709	0.07500914

7% Monthly Compound Interest Table

Month	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.005833	1.000000	1.000000	0.994200	0.994200	1.00583333
2	1.011701	2.005833	0.498546	0.988435	1.982635	0.50437924
3	1.017602	3.017534	0.331396	0.982702	2.965337	0.33722976
4	1.023538	4.035136	0.247823	0.977003	3.942340	0.25365644
5	1.029509	5.058675	0.197680	0.971337	4.913677	0.20351357
6	1.035514	6.088184	0.164253	0.965704	5.879381	0.17008594
7	1.041555	7.123698	0.140377	0.960103	6.839484	0.14620986
8	1.047631	8.165253	0.122470	0.954535	7.794019	0.12830352
9	1.053742	9.212883	0.108544	0.948999	8.743018	0.11437698
10	1.059889	10.266625	0.097403	0.943495	9.686513	0.10323632
11	1.066071	11.326514	0.088288	0.938024	10.624537	0.09412175
12	1.072290	12.392585	0.080693	0.932583	11.557120	0.08652675
Year						
1	1.072290	12.392585	0.080693	0.932583	11.557120	0.08652675
2	1.149806	25.681032	0.038939	0.869712	22.335099	0.04477258
3	1.232926	39.930101	0.025044	0.811079	32.386464	0.03087710
4	1.322054	55.209236	0.018113	0.756399	41.760201	0.02394624
5	1.417625	71.592902	0.013968	0.705405	50.501994	0.01980120
6	1.520106	89.160944	0.011216	0.657849	58.654444	0.01704901
7	1.629994	107.998981	0.009259	0.613499	66.257285	0.01509268
8	1.747826	128.198821	0.007800	0.572139	73.347569	0.01363372
9	1.874177	149.858909	0.006673	0.533568	79.959850	0.01250628
10	2.009661	173.084807	0.005778	0.497596	86.126354	0.01161085
11	2.154940	197.989707	0.005051	0.464050	91.877134	0.01088410
12	2.310721	224.694985	0.004450	0.432765	97.240216	0.01028381
13	2.477763	253.330789	0.003947	0.403590	102.241738	0.00978074
14	2.656881	284.036677	0.003521	0.376381	106.906074	0.00935401
15	2.848947	316.962297	0.003155	0.351007	111.255958	0.00898828
16	3.054897	352.268112	0.002839	0.327343	115.312587	0.00867208
17	3.275736	390.126188	0.002563	0.305275	119.095732	0.00839661
18	3.512539	430.721027	0.002322	0.284694	122.623831	0.00815502
19	3.766461	474.250470	0.002109	0.265501	125.914077	0.00794192
20	4.038739	520.926660	0.001920	0.247602	128.982506	0.00775299
21	4.330700	570.977075	0.001751	0.230910	131.844073	0.00758472
22	4.643766	624.645640	0.001601	0.215342	134.512723	0.00743424
23	4.979464	682.193909	0.001466	0.200825	137.001461	0.00729919
24	5.339430	743.902347	0.001344	0.187286	139.322418	0.00717760
25	5.725418	810.071693	0.001234	0.174660	141.486903	0.00706779
30	8.116497	1219.970996	0.000820	0.123206	150.307568	0.00665302
35	11.506152	1801.054601	0.000555	0.086910	156.529709	0.00638856
40	16.311411	2624.813398	0.000381	0.061307	160.918839	0.00621431

8% Annual Compound Interest Table

Period	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.080000	1.000000	1.000000	0.925926	0.925926	1.08000000
2	1.166400	2.080000	0.480769	0.857339	1.783265	0.56076923
3	1.259712	3.246400	0.308034	0.793832	2.577097	0.38803351
4	1.360489	4.506112	0.221921	0.735030	3.312127	0.30192080
5	1.469328	5.866601	0.170456	0.680583	3.992710	0.25045645
6	1.586874	7.335929	0.136315	0.630170	4.622880	0.21631539
7	1.713824	8.922803	0.112072	0.583490	5.206370	0.19207240
8	1.850930	10.636628	0.094015	0.540269	5.746639	0.17401476
9	1.999005	12.487558	0.080080	0.500249	6.246888	0.16007971
10	2.158925	14.486562	0.069029	0.463193	6.710081	0.14902949
11	2.331639	16.645487	0.060076	0.428883	7.138964	0.14007634
12	2.518170	18.977126	0.052695	0.397114	7.536078	0.13269502
13	2.719624	21.495297	0.046522	0.367698	7.903776	0.12652181
14	2.937194	24.214920	0.041297	0.340461	8.244237	0.12129685
15	3.172169	27.152114	0.036830	0.315242	8.559479	0.11682954
16	3.425943	30.324283	0.032977	0.291890	8.851369	0.11297687
17	3.700018	33.750226	0.029629	0.270269	9.121638	0.10962943
18	3.996019	37.450244	0.026702	0.250249	9.371887	0.10670210
19	4.315701	41.446263	0.024128	0.231712	9.603599	0.10412763
20	4.660957	45.761964	0.021852	0.214548	9.818147	0.10185221
21	5.033834	50.422921	0.019832	0.198656	10.016803	0.09983225
22	5.436540	55.456755	0.018032	0.183941	10.200744	0.09803207
23	5.871464	60.893296	0.016422	0.170315	10.371059	0.09642217
24	6.341181	66.764759	0.014978	0.157699	10.528758	0.09497796
25	6.848475	73.105940	0.013679	0.146018	10.674776	0.09367878
26	7.396353	79.954415	0.012507	0.135202	10.809978	0.09250713
27	7.988061	87.350768	0.011448	0.125187	10.935165	0.09144810
28	8.627106	95.338830	0.010489	0.115914	11.051078	0.09048891
29	9.317275	103.965936	0.009619	0.107328	11.158406	0.08961854
30	10.062657	113.283211	0.008827	0.099377	11.257783	0.08882743
31	10.867669	123.345868	0.008107	0.092016	11.349799	0.08810728
32	11.737083	134.213537	0.007451	0.085200	11.434999	0.08745081
33	12.676050	145.950620	0.006852	0.078889	11.513888	0.08685163
34	13.690134	158.626670	0.006304	0.073045	11.586934	0.08630411
35	14.785344	172.316804	0.005803	0.067635	11.654568	0.08580326
36	15.968172	187.102148	0.005345	0.062625	11.717193	0.08534467
37	17.245626	203.070320	0.004924	0.057986	11.775179	0.08492440
38	18.625276	220.315945	0.004539	0.053690	11.828869	0.08453894
39	20.115298	238.941221	0.004185	0.049713	11.878582	0.08418513
40	21.724521	259.056519	0.003860	0.046031	11.924613	0.08386016

8% Monthly Compound Interest Table

Month	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.006667	1.000000	1.000000	0.993377	0.993377	1.00666667
2	1.013378	2.006667	0.498339	0.986799	1.980176	0.50500554
3	1.020134	3.020044	0.331121	0.980264	2.960440	0.33778762
4	1.026935	4.040178	0.247514	0.973772	3.934212	0.25418051
5	1.033781	5.067113	0.197351	0.967323	4.901535	0.20401772
6	1.040673	6.100893	0.163910	0.960917	5.862452	0.17057709
7	1.047610	7.141566	0.140025	0.954553	6.817005	0.14669198
8	1.054595	8.189176	0.122112	0.948232	7.765237	0.12877907
9	1.061625	9.243771	0.108181	0.941952	8.707189	0.11484763
10	1.068703	10.305396	0.097037	0.935714	9.642903	0.10370321
11	1.075827	11.374099	0.087919	0.929517	10.572420	0.09458572
12	1.083000	12.449926	0.080322	0.923361	11.495782	0.08698843
Year						
1	1.083000	12.449926	0.080322	0.923361	11.495782	0.08698843
2	1.172888	25.933190	0.038561	0.852596	22.110544	0.04522729
3	1.270237	40.535558	0.024670	0.787255	31.911806	0.03133637
4	1.375666	56.349915	0.017746	0.726921	40.961913	0.02441292
5	1.489846	73.476856	0.013610	0.671210	49.318433	0.02027639
6	1.613502	92.025325	0.010867	0.619770	57.034522	0.01753324
7	1.747422	112.113308	0.008920	0.572272	64.159261	0.01558621
8	1.892457	133.868583	0.007470	0.528414	70.737970	0.01413668
9	2.049530	157.429535	0.006352	0.487917	76.812497	0.01301871
10	2.219640	182.946035	0.005466	0.450523	82.421481	0.01213276
11	2.403869	210.580392	0.004749	0.415996	87.600600	0.01141545
12	2.603389	240.508387	0.004158	0.384115	92.382800	0.01082453
13	2.819469	272.920390	0.003664	0.354677	96.798498	0.01033074
14	3.053484	308.022574	0.003247	0.327495	100.875784	0.00991318
15	3.306921	346.038222	0.002890	0.302396	104.640592	0.00955652
16	3.581394	387.209149	0.002583	0.279221	108.116871	0.00924925
17	3.878648	431.797244	0.002316	0.257822	111.326733	0.00898257
18	4.200574	480.086128	0.002083	0.238063	114.290596	0.00874963
19	4.549220	532.382966	0.001878	0.219818	117.027313	0.00854501
20	4.926803	589.020416	0.001698	0.202971	119.554292	0.00836440
21	5.335725	650.358746	0.001538	0.187416	121.887606	0.00820428
22	5.778588	716.788127	0.001395	0.173053	124.042099	0.00806178
23	6.258207	788.731114	0.001268	0.159790	126.031475	0.00793453
24	6.777636	866.645333	0.001154	0.147544	127.868388	0.00782054
25	7.340176	951.026395	0.001051	0.136237	129.564523	0.00771816
30	10.935730	1490.359449	0.000671	0.091443	136.283494	0.00733765
35	16.292550	2293.882485	0.000436	0.061378	140.793338	0.00710261
40	24.273386	3491.007831	0.000286	0.041197	143.820392	0.00695312

10% Annual Compound Interest Table

Period	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.100000	1.000000	1.000000	0.909091	0.909091	1.10000000
2	1.210000	2.100000	0.476190	0.826446	1.735537	0.57619048
3	1.331000	3.310000	0.302115	0.751315	2.486852	0.40211480
4	1.464100	4.641000	0.215471	0.683013	3.169865	0.31547080
5	1.610510	6.105100	0.163797	0.620921	3.790787	0.26379748
6	1.771561	7.715610	0.129607	0.564474	4.355261	0.22960738
7	1.948717	9.487171	0.105405	0.513158	4.868419	0.20540550
8	2.143589	11.435888	0.087444	0.466507	5.334926	0.18744402
9	2.357948	13.579477	0.073641	0.424098	5.759024	0.17364054
10	2.593742	15.937425	0.062745	0.385543	6.144567	0.16274539
11	2.853117	18.531167	0.053963	0.350494	6.495061	0.15396314
12	3.138428	21.384284	0.046763	0.318631	6.813692	0.14676332
13	3.452271	24.522712	0.040779	0.289664	7.103356	0.14077852
14	3.797498	27.974983	0.035746	0.263331	7.366687	0.13574622
15	4.177248	31.772482	0.031474	0.239392	7.606080	0.13147378
16	4.594973	35.949730	0.027817	0.217629	7.823709	0.12781662
17	5.054470	40.544703	0.024664	0.197845	8.021553	0.12466413
18	5.559917	45.599173	0.021930	0.179859	8.201412	0.12193022
19	6.115909	51.159090	0.019547	0.163508	8.364920	0.11954687
20	6.727500	57.274999	0.017460	0.148644	8.513564	0.11745962
21	7.400250	64.002499	0.015624	0.135131	8.648694	0.11562439
22	8.140275	71.402749	0.014005	0.122846	8.771540	0.11400506
23	8.954302	79.543024	0.012572	0.111678	8.883218	0.11257181
24	9.849733	88.497327	0.011300	0.101526	8.984744	0.11129978
25	10.834706	98.347059	0.010168	0.092296	9.077040	0.11016807
26	11.918177	109.181765	0.009159	0.083905	9.160945	0.10915904
27	13.109994	121.099942	0.008258	0.076278	9.237223	0.10825764
28	14.420994	134.209936	0.007451	0.069343	9.306567	0.10745101
29	15.863093	148.630930	0.006728	0.063039	9.369606	0.10672807
30	17.449402	164.494023	0.006079	0.057309	9.426914	0.10607925
31	19.194342	181.943425	0.005496	0.052099	9.479013	0.10549621
32	21.113777	201.137767	0.004972	0.047362	9.526376	0.10497172
33	23.225154	222.251544	0.004499	0.043057	9.569432	0.10449941
34	25.547670	245.476699	0.004074	0.039143	9.608575	0.10407371
35	28.102437	271.024368	0.003690	0.035584	9.644159	0.10368971
36	30.912681	299.126805	0.003343	0.032349	9.676508	0.10334306
37	34.003949	330.039486	0.003030	0.029408	9.705917	0.10302994
38	37.404343	364.043434	0.002747	0.026735	9.732651	0.10274692
39	41.144778	401.447778	0.002491	0.024304	9.756956	0.10249098
40	45.259256	442.592556	0.002259	0.022095	9.779051	0.10225941

10% Monthly Compound Interest Table

Month	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.008333	1.000000	1.000000	0.991736	0.991736	1.00833333
2	1.016736	2.008333	0.497925	0.983539	1.975275	0.50625864
3	1.025209	3.025069	0.330571	0.975411	2.950686	0.33890426
4	1.033752	4.050278	0.246897	0.967350	3.918036	0.25522994
5	1.042367	5.084031	0.196694	0.959355	4.877391	0.20502766
6	1.051053	6.126398	0.163228	0.951427	5.828817	0.17156139
7	1.059812	7.177451	0.139325	0.943563	6.772381	0.14765856
8	1.068644	8.237263	0.121400	0.935765	7.708146	0.12973288
9	1.077549	9.305907	0.107459	0.928032	8.636178	0.11579196
10	1.086529	10.383456	0.096307	0.920362	9.556540	0.10464038
11	1.095583	11.469985	0.087184	0.912756	10.469296	0.09551741
12	1.104713	12.565568	0.079583	0.905212	11.374508	0.08791589
Year						
1	1.104713	12.565568	0.079583	0.905212	11.374508	0.08791589
2	1.220391	26.446915	0.037812	0.819410	21.670855	0.04614493
3	1.348182	41.781821	0.023934	0.741740	30.991236	0.03226719
4	1.489354	58.722492	0.017029	0.671432	39.428160	0.02536258
5	1.645309	77.437072	0.012914	0.607789	47.065369	0.02124704
6	1.817594	98.111314	0.010193	0.550178	53.978665	0.01852584
7	2.007920	120.950418	0.008268	0.498028	60.236667	0.01660118
8	2.218176	146.181076	0.006841	0.450821	65.901488	0.01517416
9	2.450448	174.053713	0.005745	0.408089	71.029355	0.01407869
10	2.707041	204.844979	0.004882	0.369407	75.671163	0.01321507
11	2.990504	238.860493	0.004187	0.334392	79.872986	0.01251988
12	3.303649	276.437876	0.003617	0.302696	83.676528	0.01195078
13	3.649584	317.950102	0.003145	0.274004	87.119542	0.01147848
14	4.031743	363.809201	0.002749	0.248032	90.236201	0.01108203
15	4.453920	414.470346	0.002413	0.224521	93.057439	0.01074605
16	4.920303	470.436376	0.002126	0.203240	95.611259	0.01045902
17	5.435523	532.262780	0.001879	0.183975	97.923008	0.01021210
18	6.004693	600.563216	0.001665	0.166536	100.015633	0.00999844
19	6.633463	676.015601	0.001479	0.150751	101.909902	0.00981259
20	7.328074	759.368836	0.001317	0.136462	103.624619	0.00965022
21	8.095419	851.450244	0.001174	0.123527	105.176801	0.00950780
22	8.943115	953.173779	0.001049	0.111818	106.581856	0.00938246
23	9.879576	1065.549097	0.000938	0.101219	107.853730	0.00927182
24	10.914097	1189.691580	0.000841	0.091625	109.005045	0.00917389
25	12.056945	1326.833403	0.000754	0.082940	110.047230	0.00908701
30	19.837399	2260.487925	0.000442	0.050410	113.950820	0.00877572
35	32.638650	3796.638052	0.000263	0.030639	116.323377	0.00859672
40	53.700663	6324.079581	0.000158	0.018622	117.765391	0.00849146

12% Annual Compound Interest Table

Period	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.120000	1.000000	1.000000	0.892857	0.892857	1.12000000
2	1.254400	2.120000	0.471698	0.797194	1.690051	0.59169811
3	1.404928	3.374400	0.296349	0.711780	2.401831	0.41634898
4	1.573519	4.779328	0.209234	0.635518	3.037349	0.32923444
5	1.762342	6.352847	0.157410	0.567427	3.604776	0.27740973
6	1.973823	8.115189	0.123226	0.506631	4.111407	0.24322572
7	2.210681	10.089012	0.099118	0.452349	4.563757	0.21911774
8	2.475963	12.299693	0.081303	0.403883	4.967640	0.20130284
9	2.773079	14.775656	0.067679	0.360610	5.328250	0.18767889
10	3.105848	17.548735	0.056984	0.321973	5.650223	0.17698416
11	3.478550	20.654583	0.048415	0.287476	5.937699	0.16841540
12	3.895976	24.133133	0.041437	0.256675	6.194374	0.16143681
13	4.363493	28.029109	0.035677	0.229174	6.423548	0.15567720
14	4.887112	32.392602	0.030871	0.204620	6.628168	0.15087125
15	5.473566	37.279715	0.026824	0.182696	6.810864	0.14682424
16	6.130394	42.753280	0.023390	0.163122	6.973986	0.14339002
17	6.866041	48.883674	0.020457	0.145644	7.119630	0.14045673
18	7.689966	55.749715	0.017937	0.130040	7.249670	0.13793731
19	8.612762	63.439681	0.015763	0.116107	7.365777	0.13576300
20	9.646293	72.052442	0.013879	0.103667	7.469444	0.13387878
21	10.803848	81.698736	0.012240	0.092560	7.562003	0.13224009
22	12.100310	92.502584	0.010811	0.082643	7.644646	0.13081051
23	13.552347	104.602894	0.009560	0.073788	7.718434	0.12955996
24	15.178629	118.155241	0.008463	0.065882	7.784316	0.12846344
25	17.000064	133.333870	0.007500	0.058823	7.843139	0.12749997
26	19.040072	150.333934	0.006652	0.052521	7.895660	0.12665186
27	21.324881	169.374007	0.005904	0.046894	7.942554	0.12590409
28	23.883866	190.698887	0.005244	0.041869	7.984423	0.12524387
29	26.749930	214.582754	0.004660	0.037383	8.021806	0.12466021
30	29.959922	241.332684	0.004144	0.033378	8.055184	0.12414366
31	33.555113	271.292606	0.003686	0.029802	8.084986	0.12368606
32	37.581726	304.847719	0.003280	0.026609	8.111594	0.12328033
33	42.091533	342.429446	0.002920	0.023758	8.135352	0.12292031
34	47.142517	384.520979	0.002601	0.021212	8.156564	0.12260064
35	52.799620	431.663496	0.002317	0.018940	8.175504	0.12231662
36	59.135574	484.463116	0.002064	0.016910	8.192414	0.12206414
37	66.231843	543.598690	0.001840	0.015098	8.207513	0.12183959
38	74.179664	609.830533	0.001640	0.013481	8.220993	0.12163980
39	83.081224	684.010197	0.001462	0.012036	8.233030	0.12146197
40	93.050970	767.091420	0.001304	0.010747	8.243777	0.12130363

12% Monthly Compound Interest Table

Month	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.010000	1.000000	1.000000	0.990099	0.990099	1.01000000
2	1.020100	2.010000	0.497512	0.980296	1.970395	0.50751244
3	1.030301	3.030100	0.330022	0.970590	2.940985	0.34002211
4	1.040604	4.060401	0.246281	0.960980	3.901966	0.25628109
5	1.051010	5.101005	0.196040	0.951466	4.853431	0.20603980
6	1.061520	6.152015	0.162548	0.942045	5.795476	0.17254837
7	1.072135	7.213535	0.138628	0.932718	6.728195	0.14862828
8	1.082857	8.285671	0.120690	0.923483	7.651678	0.13069029
9	1.093685	9.368527	0.106740	0.914340	8.566018	0.11674036
10	1.104622	10.462213	0.095582	0.905287	9.471305	0.10558208
11	1.115668	11.566835	0.086454	0.896324	10.367628	0.09645408
12	1.126825	12.682503	0.078849	0.887449	11.255077	0.08884879
Year						
1	1.126825	12.682503	0.078849	0.887449	11.255077	0.08884879
2	1.269735	26.973465	0.037073	0.787566	21.243387	0.04707347
3	1.430769	43.076878	0.023214	0.698925	30.107505	0.03321431
4	1.612226	61.222608	0.016334	0.620260	37.973959	0.02633384
5	1.816697	81.669670	0.012244	0.550450	44.955038	0.02224445
6	2.047099	104.709931	0.009550	0.488496	51.150391	0.01955019
7	2.306723	130.672274	0.007653	0.433515	56.648453	0.01765273
8	2.599273	159.927293	0.006253	0.384723	61.527703	0.01625284
9	2.928926	192.892579	0.005184	0.341422	65.857790	0.01518423
10	3.300387	230.038689	0.004347	0.302995	69.700522	0.01434709
11	3.718959	271.895856	0.003678	0.268892	73.110752	0.01367788
12	4.190616	319.061559	0.003134	0.238628	76.137157	0.01313419
13	4.722091	372.209054	0.002687	0.211771	78.822939	0.01268666
14	5.320970	432.096982	0.002314	0.187936	81.206434	0.01231430
15	5.995802	499.580198	0.002002	0.166783	83.321664	0.01200168
16	6.756220	575.621974	0.001737	0.148012	85.198824	0.01173725
17	7.613078	661.307751	0.001512	0.131353	86.864707	0.01151216
18	8.578606	757.860630	0.001320	0.116569	88.343095	0.01131950
19	9.666588	866.658830	0.001154	0.103449	89.655089	0.01115386
20	10.892554	989.255365	0.001011	0.091806	90.819416	0.01101086
21	12.274002	1127.400210	0.000887	0.081473	91.852698	0.01088700
22	13.830653	1283.065279	0.000779	0.072303	92.769683	0.01077938
23	15.584726	1458.472574	0.000686	0.064165	93.583461	0.01068565
24	17.561259	1656.125905	0.000604	0.056944	94.305647	0.01060382
25	19.788466	1878.846626	0.000532	0.050534	94.946551	0.01053224
30	35.949641	3494.964133	0.000286	0.027817	97.218331	0.01028613
35	65.309595	6430.959471	0.000155	0.015312	98.468831	0.01015550
40	118.647725	11764.772510	0.000085	0.008428	99.157169	0.01008500