GEORGIA DEPARTMENT OF REVENUE

Course II-B

Application of the Income Approach To Value

For Educational Purposes Only:

The material within is intended to give the course participant a solid understanding of general principles in the subject area. As such, the material may not necessarily reflect the official procedures and policies of the Georgia Department of Revenue or the Department's official interpretation of the laws of the State of Georgia. The application of applicability to specific situations of the theories, techniques, and approaches discussed herein must be determined on a case-by-case basis.

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REVIEW OF DIRECT CAPITALIZATION

The income capitalization approach to property valuation, also commonly referred to as the income approach, is a method by which the fair market value of real estate is based upon the amount of net operating income (NOI) the property generates.

Specifically, the income capitalization approach uses two variables to determine a property's fair market value. First is the net operating income which refers to a property's annual income after all its operating expenses are paid. Second is the overall capitalization rate (R_O), a percentage that expresses a property's estimated NOI as a percentage of its market value.

The formula for direct capitalization is:

$$\text{Value} = \frac{\text{Net Operating Income (NOI)}}{\text{Capitalization Rate}}$$

The formula (equation) may be expressed as:

I (income) = Rate x Value

R (rate) = Income ÷ Value

V (value) = Income ÷ Rate

Reminder: "**I**" – represents the ***Annual Net Operating Income***
"**R**" – represents the ***Overall Capitalization Rate*** (R_O)
"**V**" – represents the ***Value*** (represented by a recent sale)

DETERMINING THE NET OPERATING INCOME

Determining the Net Operating Income involves a process known as "Reconstruction of the Owner's Operating Statement". Often, as the appraiser is developing the NOI financial information is provided that must be restated (reconstructed) for the appraisal process.

The reconstructed income & expense statement (operating statement) may be developed from rent comparables in the area. Regardless of the source, the resulting NOI should be developed from market data and representative of conditions relating to the valuation date.

The process of reconstructing the operating statement is as follows:

POTENTIAL GROSS INCOME

-Vacancy & Collection Loss

+Miscellaneous Income

=EFFECTIVE GROSS INCOME

-Allowable Expenses

(Operating Expenses & Reserves for Replacement)

=NET OPERATING INCOME

DEFINITIONS:

Potential Gross Income is the annual economic rent for the property at100 % occupancy. It includes the market rent from all scheduled sources.

Vacancy and Collection Loss

Properties will experience some amount of vacancy during their economic life.

During the economic life of a property, there will be a loss of rental income because of the failure of tenants to pay the rent.

The amount that is allowed for vacancy and collection loss must be based on an analysis of properties that are comparable to the subject.

Miscellaneous Income (Miscellaneous income may come from several sources other than the scheduled rents.)

Miscellaneous income covers all income generated by the operation of the real estate that is not derived directly from scheduled rental of space. Some examples of miscellaneous income are parking fees, vending machines,

coin operated laundries, and cable installation. Miscellaneous income included the real estate valuation must be attributable to the real estate.

Effective gross income (EGI)

Effective gross income is obtained after determining the potential gross income for the property. EGI is Potential Gross Income less vacancy and collection loss plus appropriate miscellaneous income.

Proper expenses (deducted)

Operating expenses are ordinary and typical expenses that are necessary to keep the property functional and rented competitively with other properties in the area. Operating expenses vary from property to property, depending on the type of occupancy, use type, and quality of management.

In analyzing the operating expenses for a property, the operating statements from comparable properties must be reviewed and the following considered:

Does the expense amount appear to be typical for the property and is the amount verified by expense statements of comparable properties?

Do the expenses tend to appear infrequently?

Do the expenses appear to indicate typical management?

Are the various reported expenses consistent as they relate to each other (maintenance, age, and reserves for replacement)?

Is the ratio of expenses to effective gross income comparable to those for competitive properties?

Some Examples of proper expenses are:

- Management
- Insurance
- Utilities
- Maintenance
- Legal and accounting fees

- Employee wages and fringe benefits
- Lawn maintenance
- Miscellaneous
- Reserves for replacement

Reserves for replacement

Short-lived items are items which are expected to have a remaining economic life less than the remaining economic life of the property. The amount of expense for short-lived items is computed by dividing the replacement cost of the item by the number of years of total economic life. Some examples are carpets, drapes, heating, air conditioning, ranges, refrigerators, water heaters, and paving, etc.

Types Of Rents

Market rent - The rental income that a property would most probably command in the open market; indicated by current rents paid and asked for comparable space as of the date of the appraisal.

Market rent should reflect the location of the property, size of the property, supply and demand factors, and terms of lease between knowledgeable and prudent owners and tenants and is also known as economic rent.

Contract rent - The actual rental income specified in a lease; actual rent payable as distinguished from market rent or rent that could be expected if the property were available for rent on the open market.

Minimum base rent - The fixed portion of the rent under the terms of a percentage lease.

Percentage rent - Percentage rent is rental income received in accordance with the terms of a percentage clause in a lease. Percentage rent is typically derived from rental tenants and is based on a certain percentage of their sales revenue.

Overage rent - the percentage rent paid over and above the guaranteed minimum rate; payable under the terms of a lease that requires additional rent under prescribed conditions such as the achievement of a particular level or profit of

retail sales volume by the tenant; commonly used in connection with additional rent paid under a percentage clause.

Deficit rent (leasehold income) - Occurs when market rent is greater than contract rent.

Excess rent - the amount of which contract exceeds market rent at the time of the appraisal; created by a favorable lease and may reflect locational advantage, unusual management, or lease execution in an earlier, stronger rental market. Due to higher risk inherent in the receipt of excess rent, it may be calculated separately and capitalized at a higher rate in the income capitalization approach.

Types of Rentable Area

Gross building area - The total constructed area of a building as measured on the outside of the structure.

Gross Leasable area (GLA) -All area within the outside walls, including lobbies, washrooms, janitor's closets, and so on, but excluding building stairs, fire towers, elevator shafts, flues, vents, stacks, pipe shafts, and vertical ducts if they serve more than one floor.

Net Leasable area (NLA) - The area within a building or structure that is occupied by an individual tenant. Net Leasable area does not include any of the common areas, such as lobbies and restrooms, shared by other tenants and is also referred to as net rentable area (NRA).

Example 1-1 Reconstruction of the Income and Expense Statement

You have been given the owner's operating statement for a 60-unit apartment complex. The statement has been prepared by the owner's accountant for income tax purposes and you will need to reconstruct it for appraisal purposes.

Income and Expense Statement
Dove Tree Apartments

Income		
Apartment rents		$581,400
Laundry facilities		7,500
	Gross income	$588,900
Expenses:		
	Real estate taxes	$45,450
	Insurance	30,600
	Salaries	34,500
	Fringe benefits	9,650
	Painting 10 units	20,000
	Utilities	73,100
	Grounds maintenance	18,500
	Advertising	4,800
	Depreciation	195,000
	Debt service (principal & interest)	198,400
	Replace 5 refrigerators	4,000
	Replace 8 stoves	5,600
	Replace 10 water heaters	6,000
Total expenses:		**$645,600**

An analysis of the market indicates a potential gross income (PGI) of $630,000, and a rate of 5% for vacancy and collection loss. The owner's miscellaneous income is typical, and the allowable expenses are similar to comparable complexes requiring no adjustment. Also, expenses for items that are considered reserves for replacement are comparable to similar complexes. The owner did not include any allowance for management. However, a management fee of 5% of the effective gross income is an appropriate adjustment.

The stoves and refrigerators have a 15-year life, while the water heaters have a 10-year life. There is one stove, refrigerator, and water heater in each apartment. The roof cover for the complex has a 20-year life and would cost $60,000 to replace. All units should be repainted on a five-year cycle. The floor covering in each unit lasts 9 years and costs $1,200 per unit to replace.

Using market data and the owner's allowable expenses you can now reconstruct the income and expense statement.

Example 1-1 Solution

Income and Expense Statement
Dove Tree Apartments

Potential Gross Income			630,000	100.00%
Vacancy and Collection Loss			(31,500)	5.00%
Miscellaneous Income			7,500	
Effective Gross Income			606,000	
Operating Expenses				
Management	30,300	5.00%		
Insurance	30,600	5.05%		
Salaries	34,500	5.69%		
Fringe Benefits	9,650	1.59%		
Utilities	73,100	12.06%		
Grounds Maintenance	18,500	3.05%		
Advertising	4,800	0.79%		
Reserves for Replacement				
Refrigerators [(800 x 60) / 15]	3,200	0.53%		
Stoves [(700 x 60) / 15]	2,800	0.46%		
Water heaters [(600 x 60) / 10]	3,600	0.59%		
Painting [(2,000 x 60) / 5]	24,000	3.96%		
Floor cover [(1,200 x 60) / 9]	8,000	1.32%		
Roof cover (60,000 / 20)	3,000	0.50%		
Total Expenses			(246,050)	40.60%
Net Operating Income			359,950	59.40%

Problem 1-1

You are appraising a small retail shopping center and have completed a market study and expect the property to rent for $18.00 per square foot of net leasable area. The building's gross area is 80,000 square feet and the common area is 12% of the gross area.

Annual anticipated operating expenses are listed below.

Insurance.. $30,000
Depreciation…………………………………………$80,000
Management .... 4% of effective gross income
Repairs.. $22,000
Mortgage principal………………………………..$42,000
Legal and accounting.......................... 1% of effective gross income
Miscellaneous expenses $5,600
Mortgage interest…………………………………..$328,000
Utilities,,……………$98,200
Real estate taxes .... ……………..$50,000

Vacancy and collection loss is expected to be 10% and there is no miscellaneous income.

Reserves for replacement includes replacement of store fronts, painting and decorating, and floor covering replacement and the total is confirmed as $130,000.

What is the Annual Net Operating Income (NOI)?

Rent Concessions

A rent concession is a loss in income for the owner because of an agreement with the tenant to either reduce the rent on short term basis, or allow free rent for a short term, or supply higher than normal tenant improvement (TI) work.
The concession calculation and subsequent deduction can only be made if this type of concession is a standard market practice. It may be that the owner has a need to fill vacant space as soon as possible, and elects to handle it by granting concessions, when in fact, the typical landlord does not. Adjusting for atypical rent concession will provide a value estimate of the leased fee value, not fee simple value.

The appraiser must research the current market regarding concessions before making decisions on when, or if, it is appropriate to adjust the income stream.

If the rent concession is considered an appropriate deduction because of what the market indicates, it should be removed from the income in a manner like a vacancy deduction. However, if the concession being given appears to be short-term in nature, a suggestion for a more appropriate way to address that income loss would be to deduct the lost rent by discounting it over the projected short term, then deducting that total from the final income estimate.

Example 1-2
The lease on an office building is for five years with a base rate of $30.00 per square foot. The landlord is offering a rent concession of three months free rent. What would be the effective rent amount per square foot?

Rent concession term		3 months
Lease tern		5 years (60 months)
Concession percentage	3 months / 60 months	0.05
Base rent		$30.00 / square foot (SF)
Rent concession per square foot	$30.00 x 0.05	$1.50 per SF
Effective rent	$30.00 SF - $1.50 SF	$28.50 SF

Problem 1-2

Due to supply demand for this property type the owner must give a rent concession (market required) to lease a property. The annual rent spelled out in the lease is $1,000 per month plus one-month free rent. In this situation, what is the effective rent per month?

EXPENSE REIMBURSEMENTS & CAM EXPENSES, PASS THROUGH EXPENSES & EXPENSE STOPS

Expense reimbursements:

When a tenant is forced to pay the landlord additional expenses because of the expense stop, that amount of money is normally classified as expense reimbursements and shows up as income in the operating statement.

The expense portion of the operating statement will probably reflect all expenses paid by the landlord, including that amount reimbursed. The appraiser needs to use care when using this information when analyzing a specific property's income and expense data.

Reimbursements may include common area maintenance, CAM (see below) expenses being paid by the tenant. The operating statement should be checked to see if these expenses are showing up as an expense item or not.

CAM Expenses

CAM (common area maintenance) expenses are usually called out in a lease for the purpose of defining a specific cost that the tenant will be responsible for. This type of "called out" expense is more common with retail leases than other property type. Newer retail property tenants will ordinarily pay all rented space expenses. This would leave the landlord paying all operating costs for the common area. Many landlords will pass this expense on to the tenant by setting that item up in the lease as something the tenant must pay the landlord, usually on a prorated basis (tenant space to total area ratio).

Pass-through Expenses

This is a term used for those expenses the property incurs that exceed the level of expense the landlord is willing to pay. This expense is then paid by the tenant, usually on a prorated basis according to its space percentage of the total area.

Expense Stop(s)

This is a term used for the level of expense that the landlord is willing to pay. This number is usually expressed as a dollar amount per square foot, and includes all expenses that the landlord is responsible for in the year the lease was written.

Sometimes the owner will use the following year as the base year for the stop, and project the expenses for that period. Many times the term "expense stop" will not be used. Instead, the landlord might choose to call the expense limit the "base year expenses".

This number is usually the total dollar amount of the landlord's expenses for the year considered the base year.

Leasing Commissions

Many times, owners of multi-tenant properties will turn to leasing agents to secure leases for their building, including lease preparation, and sometimes, property management.

The leasing agents make their money by charging the landlord a fee or commission for securing the tenant. That fee is usually based on the total revenue generated by the lease they secure. It is usually a small percentage somewhere in the 5% to 7% range.

Leasing commissions may be paid up front, or may be paid out monthly/annually over the life of the lease. This is a real expense to many owners, and could be an appropriate operating expense deduction.

Care should be taken when using this cost for mass appraisal to ensure that a common or consistent rate is used for all property types where it is applicable.

Tenant Improvements

Tenant improvement costs are a capital expenditure that can be annualized and recovered by the landlord through the rent payment. Therefore, it can be considered an appropriate annualized expense deduction.

> The problem with using tenant improvements as expenses is the variety and extent of work done from one property to another. Some older properties aren't updated with new tenant improvements to attract new tenants. Others and new building owners might spend amounts from $5 a square foot upwards to $40 a square foot.
>
> As stated, this is an appropriate expense. However, it is very difficult to standardize this expense for mass appraisal use. Care should be taken when developing the expense models if this cost is to be included.
>
> If the tenant improvements recovery costs are included in the rent, an alternative to removing those costs on an annualized basis as a expense, would be to consider them as a one-time capital expenditure and subtract that cost from the final income estimate of value. This amount would be the total TI costs necessary to bring non-current leased space up to market level rent as of the appraisal date.
>
> It would be inappropriate to expense out or subtract a lump sum cost for TI if the capitalization rate you are using wasn't developed in the same manner. This means that if the TI costs weren't removed from the income as an expense or subtracted from the adjusted sale price used in calculating the capitalization rate, then you cannot make that adjustment to your NOI or appraised value.

Example 1-3

RENT CONCESSIONS

METHOD 1:

Take a percentage of the total income from the potential gross income (PGI).

As an example: 5% of the revenue was lost due to concessions.

Multiply 5% times the PGI to get that loss in revenue.

PGI = $34,800 x 0.05 = $1,740 rent concession

METHOD 2:

Reduce the rent rate to an effective rent rate considering the reduced or free rent or higher than market TI allowance.

The tenant was given 3 months free rent on a 5-year (60 month) lease. The gross rent in the lease is $20.00 per SF. You can calculate effective rent by dividing 3 by 60 to get 5% and then deduct 5% from the original rental rate of $20 to get an effective rate of $19 for the life of the lease.

3 / 60 = 0.05 Percent of lease than was given as a concession.

0.05 x $20 per SF = $1.00

$20.00 - $1.00 = $19.00 Effective rent considering the concession.

If the property had 9,000 SF, the rent concession would be:

9,000 x $1 x 5 years = $45,000.

Example 1-4

RENT CONCESSIONS

Using the information for Method 2 on Example 1-3, as shown below, what would be the rent loss if the rent for Years 4 & 5 increased to $22 per SF?

The tenant was given 3 months free rent on a 5 year (60 month) lease. The gross rent in the lease is $20.00 per SF. You can calculate effective rent by dividing 3 by 60 to get 5% and then deduct 5% from the original rental rate of $20 to get an effective rate of $19 for the first three years of the lease. The effective rent for Years 4 & 5 would be $20.90 or a rent loss of $1 .10 per SF.

As this is a 5-year lease, it may be appropriate to discount the rent concession over the life of the lease. For this example, the rate used for discounting will be 9%.

Solving with Compound Interest Tables

See the annual tables in the Appendix - PV of $1 and PV of $1 per period				
$9,000	x	2.531295	=	$22,781.6550
$9,900	x	0.708425	=	$7,013.4075
$9,900	x	0.649931	=	$6,434.3169
		PV of Rent Loss		$36,229.3794

Problem 1-3

RENT CONCESSIONS

1. A property owner has given a tenant 6 months of free rent on a 6-year lease. If this is a gross lease at $24 per SF and is for 4,200 SF, what is the rent concession not considering the net present value of the rent loss?

2. What would be the rent concession for the above property if there were step-ups in the lease as listed below and a discount rate of 9%? Years 3 & 4 rent to be $25 per SF Years 5 & 6 rent to be $26.50 per SF?

Example 1-5

PASS-THROUGH EXPENSES

An office building was recently remodeled and has signed new leases with the tenants. The new leases state that all expenses above the base year annual expenses of $365,000 will be pass-through to the tenants based upon the net leasable area per tenant. Annual expenses for the next year were $392,000. If tenant A has 12,000 SF of the total NLA of 60,000 SF, what would be the amount of pass-through expense to tenant A and what is the expense stop per SF?

$392,000 - $365,000 = $27,000 of pass-through expenses

12,000 / 60,000 = 0.20 20% of the space for tenant A

$27,000 x 0.20 = $5,400 Pass-through expense for tenant A

$365,000 / 60,000 = $6.08 per SF expense stop

Problem 1-4

PASS-THROUGH EXPENSES

Base year expenses are $651,160 and current year expenses are $702,543. You are leasing 4,000 SF of the total NLA of 85,000. What is the amount of your pass-through expenses and what is the expense stop per SF?

Example 1-6

TENANT IMPROVEMENTS (TI)

Subject has 10,000 SF of NLA and was renting for $18.25 per SF. lt is anticipated that the TI costs will be $185,450 in order to attract a new tenant for a 5-year lease. What would the rent be after making the tenant improvements?

			TI	**$ 185,450**
			Size	**10,000**
		TI per Sq Ft.		**$ 18.55**
		Assuming 5 Year Lease		**$ 3.71**
		Old Rent		**$ 18.25**
Rent Necessary to Recover TI				**$ 21.96**

Problem 1-5

TENANT IMPROVEMENTS (TI)

To fill the vacant space, a property owner will have to make TI of $426,500 to the NLA of 14,200 SF. The previous tenant was paying $23.00 per SF and was considered market. If the new lease will be for 6 years, what will be the necessary rent to recover the cost of the TI?

REVIEW OF CAPITALIZATION RATES

Overall Capitalization Rate (R_O) -

Expresses the relationship between net operating income and market value of property (The overall capitalization rate represents the percentage that net operating income is in relation to total property value.)

Used in direct capitalization (IRV)

- The overall capitalization rate (R_O) is a weighted average of the mortgage capitalization rate (R_M) and the equity capitalization rate (R_E)

- Theoretically composed of the 'return on' rate (or overall yield rate Y_O or property discount rate) and a portion of the 'return of (or recapture rate) for the improvements. While all the component parts are included in the overall capitalization rate, there is not necessarily any separate identification of each of the rates included within the overall capitalization rate.

- It does not distinguish between land rate and improvement rate components. The overall capitalization rate (R_O) is a weighted average of the land and improvement capitalization rates used in straight-line capitalization. While all the component parts land capitalization rate (R_L) and improvement capitalization rate (R_B) are included in the overall capitalization rate, there is not necessarily any separate identification of each of the rates included within the overall capitalization rate.

There are six methods which can be used to derive an overall capitalization rate (R_O), they are:

- Market comparisons
- Band of investment (mortgage & equity components)
- Debt coverage ratio
- Net income ratio
- Band of investment (land & improvement components)
- Yield change techniques

Market Comparison Method

The market comparison method formula is $R_O = I \div V$

Problem 1-6

You are appraising an apartment complex and want to develop an overall rate from the market. You find six sales of similar apartment complexes. The following is the data for the six comparables:

Sale No.	Sale Price	Remaining Economic Life	Land to Improvement Ratio	Expense Ratio	Net Operating Income
1	990,000	18	1 : 4	34%	64,600
2	980,000	20	1 : 5	27%	64,000
3	920,000	20	1 : 4	30%	64,800
4	912,000	20	1 : 4	30%	64,000
5	955,000	20	1 : 4	30%	67,000
6	950,000	22	1 : 5	30%	85,500

The subject property has a remaining economic life of 20 years, a land-to-improvement ratio of 1:4 and an expense ratio of 30%. The six comparables and the subject property are of similar construction and condition.

What is the indicated overall capitalization rate by market comparison, based on the six comparable sales?

Band of Investment (mortgage & equity components)

The overall capitalization rate (R_O) is the weighted average of the annual mortgage constant (mortgage capitalization rate R_M) and equity capitalization rate (R_E), weighted by the respective proportions of total property value that each represents.

Note: This method does not include Real Estate Taxes in the R_O.

Assume typical mortgage financing requires an 8% loan with monthly payments for 20 years. Looking up the mortgage annual constant in the compound interest tables for this loan indicates a rate of 0.100373. The equity capitalization rate (R_E) is 12%. Also assume that the debt represents 60% of value and the equity represents 40% of value.

Note: The annual mortgage constant (R_M) is the ratio of the total mortgage payments for the year (annual debt service) divided by the amount of money borrowed. $R_M = I_M \div MA$

To determine the overall capitalization rate (R_O) using the band-of-investment method with debt and equity components is shown below:

Financial Components	Percent of Investment		Rate		Product
Debt	0.60	x	0.100373	=	0.060224
Equity	0.40	x	0.120000	=	0.048000
Totals	1.00		**Overall Rate (R_o)**	=	**0.108224**

Problem 1-7

Development of an R_O – Band of Investment Method Using Mortgage and Equity Components

You have been asked to develop an overall capitalization rate (R_O) for use in a commercial appraisal. You believe that similar commercial properties are financed with 70% debt and 30% equity. From discussions with investors and lenders, you have determined investors require an equity capitalization rate (R_E) of 6% for properties such as the subject and lenders require an 6% yield on loans of this type of mortgage, financed for 20 years with monthly payments

From this information develop the overall capitalization rate (R_O) applicable to the subject commercial property. *(Round your answer to 3 decimal places.)*

Equity Capitalization Rate

Equity dividend rate - Uses before-tax cash flow and is often referred to as cash-on-cash return.

Calculated by (net operating income - debt service) / equity investment.

The equity investment is the amount of money that the investor must contribute to purchase a property.

The equity dividend rate is also called the equity capitalization rate.

Internal Rate of Return (IRR) - The annualized yield rate on capital that is (or could be) generated by an asset or within a group of capital assets over a period of ownership. Internal Rate of Return is the rate that discounts all future cash flows to a net present worth equal to the original investment.

Example 1-7

Equity Dividend Rate

The Equity Dividend Rate, sometimes called the cash-on-cash return is expressed as a percentage. Suppose an investor purchased a property with a NOI of $110,000. She paid $200,000 down and has annual debt service of $80,298. What is the equity dividend rate?

Cash Down PMT		200,000
NOI		110,000
Annual Loan PMT		80,298
Annual Equity Cash Flow		29,702
	Equity Dividend Rate	0.1485

Problem 1-8

A property sold for $6,000,000 and has a NOI of $570,000. A first mortgage is available at 70%, payments are monthly over 20 years, and the interest rate is 6%.

What is the mortgage capitalization rate (R_M)?

What is the equity dividend rate (R_E)?

Example 1-8 Debt Coverage Ratio

R_O, Debt coverage ratio method,

Often used by lending institutions that are sensitive to the safety of their loan investments. The lenders are concerned with profits expected by the property and providing a safety margin so that the borrower will be able to meet debt service obligations on the amount loaned on the property. It is a measure of solvency.

The formula for the DCR method is: $\mathbf{R_O = DCR \times M \times R_M}$ where DCR equals the debt coverage ratio, M equals the loan-to-value ratio, and R_M equals the mortgage capitalization rate.

Formula for developing the debt coverage ratio (DCR) is:

NOI = Net Operating Income

I_M = Income necessary to satisfy mortgage payments (**annual debt service**)

Note: can also be computed by multiplying R_M times the amount borrowed.

Assume a property's net operating income is $700,000 and the annual debt service is $511,740. The debt coverage ratio is calculated as follows:

$$DCR = \$700,000 \div \$511,740 = 1.3679$$

If a mortgage can be obtained to finance 75% of the value of the property with an annual mortgage requirement of 11.19% of the amount financed, the overall rate is calculated as follows:

$$\begin{aligned} R_O &= 1.3679 \times 0.1119 \times 0.75 \\ &= 0.1148 \\ &= 0.115 \text{ (rounded)} \end{aligned}$$

Problem 1-9

Debt Coverage Ratio

A property with an annual net operating income of $62,000 recently sold for $700,000. It was purchased with a mortgage. The loan terms called for a loan to value ratio of 70% with an interest rate of 8%. The loan requires monthly payments over a term of 20 years.

Given this information calculate the overall capitalization rate (if possible).

What method, if any, did you use to determine the R_O?

What is the debt coverage ratio?

Example 1-9 Net Income Ratio Method

The overall capitalization rate (R_o) can be developed by dividing the net income ratio by the effective gross income multiplier. The net income ratio is the ratio of net operating income to effective gross income. The effective gross income multiplier is the total sales price divided by the annual effective gross income.

For example, assume the effective gross income for a commercial property is $234,000 and the operating expenses for similar properties typically amount to 40 % of effective gross income. Also assume the property sold recently for $1,123,200. What is the overall capitalization rate?

The formula for determining the overall capitalization rate (Ro) using the net income ratio method is:

$$\mathbf{R_o} = \frac{\text{NIR (Net Income Ratio)}}{\text{EGIM (Effective Gross Income Multiplier)}}$$

NIR = Net Income Ratio
EGIM = Effective Gross Income Multiplier

The net income ratio is 0.60 which is obtained by deducting the operating expense ratio of 40% from 100%. The effective gross income multiplier of 4.80 is developed by dividing the sales price of $1,123,200 effective gross income of $234,000. With this information the overall capitalization rate (R_o) can be computed as follows:

$$\mathbf{R_o} = \frac{0.60}{4.80} = 0.125 \text{ or } 12.5\%$$

Problem 1-10

Net Income Ratio

Using the following market data, answer questions A through G.

An apartment complex's potential gross income is $180,000. Vacancy and collection losses are projected to be 6%. Annual total operating expenses are forecast to be $59,800. The property recently sold for $ 1,000,000.

A. What is the potential gross income?

B. What is the effective gross income?

C. What is the net operating income?

D. What is the operating expense ratio?

E. What is the net income ratio?

F. What is the effective gross income multiplier?

G. What is the overall capitalization rate?

Example 1-10 Band of Investment (Land and Building) Method

The overall capitalization rate (R_O) is the weighted average of the land and improvement capitalization rates, weighted by the respective proportions of total property value that each represents. For example, assume the land capitalization rate (R_L) is 10% and the improvement capitalization rate (R_B) is 14%. Assume further that the land represents 25% of value and the improvements represent 75% of value.

To determine the overall capitalization rate (Ro) using the band-of-investment method, proceed as follows:

Property Component	Percent of Investment		Cap Rate		Product
Land	0.25	x	0.1000	=	0.0250
Improvement	0.75	x	0.1400	=	0.1050
Totals	1.00				0.1300

Note: The land capitalization rate does not include a recapture rate. Land is a non-wasting asset and does not depreciate.

Problem 1-11

Band of Investment (Land and Building)

You are developing an overall capitalization rate for a property. After interviewing investors, you believe that a return of 8% is appropriate. The improvement is new and is believed to have a remaining economic life of 50 years. The effective tax rate in the jurisdiction is 1%.

Land represents 25% of the total property value.

What is the overall capitalization rate (R_O)?

Example 1-11 Yield Change Method

The overall capitalization rate (R_O) can be developed from the yield change formula. This methodology converts the property's overall yield rate (Y_O) (also known as the property's discount rate) to an overall capitalization rate (R_o)- The overall capitalization rate by the yield change formula is as follows:

Formula: $R_o = Y_o - CR$

This formula can be rewritten to determine an overall yield rate from the market given appropriate sales data.

Formula: $Y_o = R_o + CR$

Problem 1-12

Yield Change Formula

Scenario 1:

You are gathering market data information to appraise a strip mall in your area. After discussions with investors in local strip malls, you learned that the overall property yield (Y_o) requirement for this type of investment is 15%. Further, you have determined that typically, similar strip malls expect to have their income and value increase at a constant rate of 3% annually over the next 5 years.

What overall capitalization rate (R_o) can be extracted from this information?

Scenario 2:

A property recently sold for $800,000, with an expected annual net operating income of $64,000 during the first year of ownership. The buyer expected an increase in income and value of 3% per year over the next 5 years.

What overall yield rate (Y_o) can be extracted from this sale?

Problem 1-13

Development of an Overall Capitalization Rate – Various Methods
Given the following information:

Sales Price	$6,000,000
Land Value	$1,250,000
First Mortgage (60% of total value)	60%
Equity Cap. Rate	12.50%
Net Operating Income	$660,000
Annual Mortgage Constant (R_M)	10%
Effective Gross Income	$1,200,000
Operating Expense Ratio	45%

Compute the overall capitalization rate (R_O) by using:

- **A.** Market comparison (IRV)
- **B.** Debt coverage ratio method
- **C.** Net income ratio method
- **D.** Band of investment method (mortgage & equity components)

REVIEW QUESTIONS

1. Annual economic rent for the property at 100 percent occupancy is _____ _____ _____.

2. The rental income that a property would most probably command in the open market is _____ _____ .

3. The percentage rent paid over and above the guaranteed minimum rate is _____ _____ .

4. The amount of which market rent exceeds contract rent at the time of the appraisal is _____ _____ .

5. A loss in income for the owner because of an agreement with the tenant to either reduce the rent on short term basis, or allow free rent for a short term, or supply higher than normal tenant improvement (Tl) work is called a _____ _____ .

6. A property owner has given a tenant 6 months of free rent on a 5-year lease. If this is a gross lease at $20 per SF and is for 6,000 SF, what is the rent concession not considering the net present value of the rent loss? _____

7. _____ is a term used when a property has a vacancy rate greater than the market vacancy rate as of the date of the appraisal.

8. The tenant may have to pay CAM expenses. What does CAM mean? _____ _____ _____

9. Base year expenses are $415,650 and current year expenses are $522,755. You are leasing 5,000 SF of the total NLA of 65,000. What is the amount of your pass-through expenses? _____

10. A property sold to an investor requiring a $300,000 down payment. The property's NOI is $48,000. Based on the mortgage terms the principal and interest totals $21,000 per year. What is the cash-on-cash return? _____

11. _____ _____ costs may be necessary to rent space to a new tenant and are a capital expenditure that can be annualized and recovered by the landlord through the rent payment.

12. Subject has 14,000 SF of NLA and was renting for $20.25 per SF. It is anticipated that the TI costs with be $235,450 in order to attract a new tenant for a 5-year lease. What would the rent be after making the tenant improvements? _____

13. A property recently sold for $1,000,000. Financing terms required monthly payments at an interest rate of 6% for 20 years and a loan to value ratio of 70%. The expected NOI is $72,216. The debt coverage ratio is _____ .

14. A property has a net income ratio of 70%, the operating expense ratio is _____ .

15. If reserves are included in the expenses, the overall rate would be _____ .

Section 2

Yield Capitalization

DISCOUNTED CASH FLOW

From a quantitative viewpoint, investing in real estate is to some extent like investing in stocks. In order to profit in real estate investments, investors must determine the value of the properties they buy and make educated estimates about how much profit these investments will generate, whether through property appreciation, rental income, or a combination of both.

Introduction to Yield Capitalization

Yield capitalization converts a series of future cash flows over time into an indication of value. Yield capitalization is used to convert the future cash flows into present value by applying an appropriate yield rate. The application of capitalization rates that reflect an appropriate yield rate, the use of present value factors, and discounted cash flow analysis are all yield capitalization procedures. The present value factors used in yield capitalization are derived from compound interest formulas and calculations. These formulas and calculations are the basis of the compound interest tables (Time Value of Money)

Compound interest tables express the time value of money, a dollar received today is worth more than a dollar to be received in the future

Effective interest rate vs. nominal interest rate

The effective interest rate is the nominal interest rate divided by the number of conversion periods in one year. For example, if the nominal interest rate is 0.09, the effective interest rate would be 0.0075 for monthly compounding. (0.09 / 12 = 0.0075)

The effective interest rate is used in all calculations for the compound interest tables.

Time value of money involves six interest functions (six functions of a dollar) which measure present worth and future worth.

1. **Future worth of $ 1** shows the growth of a single deposit over a specific time period and is the reciprocal of the present worth of $ 1
2. **Future worth of $ 1 per period** shows the growth at compound interest of a series of deposits and is the reciprocal of the sinking fund factor

3. **Sinking fund factor** shows the amount of periodic payment necessary to accumulate to a specific sum and is the reciprocal of the future worth of $ 1 per period

4. **Present worth of $ 1** shows the present worth of a single future payment and is the reciprocal of the future worth of $1

5. **Present worth of $ 1 per period** shows the present worth of a series of future level income payments (an annuity) is also known as the annuity factor or the Inwood coefficient and is the reciprocal of the partial payment factor

6. **Partial payment factor** shows the periodic payment necessary to amortize a loan at a specific interest rate over a specific period of time is also known as the installment to amortize $1 and is the reciprocal of the present worth of $1 per period

Example 2-1 Using the Compounding and Discounting
Column #1 - (Future Worth of $1)

Example: A single deposit of $5,000 is deposited in a savings account which pays six% annually. If the deposit is left in the account for five years, to what amount would this investment grow?

Solution $5,000 x 1.338226 = $6,691.13

Column #2 - (Future Worth of $1 per Period)

Example: An annual deposit of $2,000 is deposited in a savings account which pays six% annually. If no principal or interest is withdrawn, how much will be in the savings account at the end of five years?

Solution $2,000 x 5.637093 = $11,274.19

Column #3 - (Sinking Fund Factor)

Example: What annual deposit at six% interest would be required to accumulate an amount of $80,000 in 20 years?

Solution $80,000 x 0.027185 = $2,174.80

Column #4 - (Present Worth of $1)

Example: On your 30th birthday you will collect the proceeds from a small insurance policy with a cash-in value of $50,000. If your 27th birthday is today, what is the present worth of your right to redeem the insurance policy assuming an annual yield of 8%?

Solution $50,000 x 0.793832 = $39,691.61

Column #5 - (Present Worth of $1 per Period)

Example: A small property you own produces an annual net income of $25,000. Assuming a rate of return of six% and a remaining economic life of 12 years, what is the present worth of this level, terminal income stream?

Solution $25,000 x 8.383844 = $209,596.10

Column #6 - (Partial Payment Factor)

Example: What is the annual payment on a mortgage of $350,000 at 6% interest over a 20-year term?

Solution $350,000 x 0.087185 = $30,514.75

Assuming month payments what is the monthly payment and annual debt service?

Solution $350,000 x 0.007165 = $2,507.51 x 12 = $30,090.12

Exercise 2-1

Application of the Compound Interest Tables (See Appendix)

You believe at the end of a lease that a property you are appraising will be worth $500,000. The lease terminates 10 years from today. What is the present value of the reversionary interest in the property today at an overall yield rate of 8%?

Exercise 2-2

Compound Interest Tables – Present Worth of an Annuity (See Appendix)

You won a lottery jackpot of $20,000,000. Your prize will be paid in the amount of $1,000,000 per year for the next 20 years (payments to be received at the end of each year).

You also have an opportunity to purchase a small business for $9,500,000 and need the money right away to close the deal. The ‘cash option is available for a one-time payout at a discount rate of 8%. If you opt for the one-time payment will you be able to purchase the small business?

Example 2-2

You have been asked to appraise a retail commercial property which is located in a mature neighborhood. The property is currently rented at market rent, which produces $272,000 of net operating income annually. The lease runs for seven more years and at the end of the lease, the property is expected to be worth $4,000,000. Assuming an appropriate overall yield rate (Y_o) of 10%, what is the present value of the property?

Solution:

Annual Net Operating Income	$272,000
Present Worth of $1 per period factor for 7 years	x4.868419
Present Worth of Annual Net Operating Income	**$1,324,210**
Future Value of Property in 7 Years	$4,000,000
Present Worth of $1 factor for 7 Years	x0.513158
Present Worth of Reversion	**$2,052,632**
Total Present Worth of Leased Property	**$3,376,842**

	Net Income		Discount Rate 10%		Present Value
7 Year Annuity	272,000	x	4.868419	=	1,324,210
Reversion	4,000,000	x	0.513158	=	2,052,632
					3,376,842

Exercise 2-3

Compound Interest Tables – Valuation of a Level Annuity with Reversion (See Appendix)

An office building is currently leased at a economic rent which produces $50,000 of net operating income annually. The lease runs for 8 more years and at the end of the lease the property is expected to be worth $3,500,000. Assuming an appropriate yield rate of 6%, what is the present value of the property?

Exercise 2-4

Valuation of Variable Annuity with Reversion (See Appendix)

You are appraising an apartment complex that is expected to produce net operating income as follows:

Year 1	$315,000
Year 2	318,000
Year 3	335,000
Year 4	352,000
Year 5	350,000

At the end of the fifth year, the property is expected to be sold for $2,000,000. From a market analysis, you have determined that the typical investor would require a return on his investment of 8% for this type of property. What is the indicated market value of the property?

DIRECT CAPITALIZATION v. DISCOUNTED CASH FLOW

To estimate value with direct capitalization, a property's expected net operating income (NOI) is divided by the overall capitalization rate. (See Fig. 2-1) Estimating value with DCF analysis requires estimates of each year's NOI along with the property's expected reversion value at the end of the holding period. The appraiser normally uses income capitalization to estimate the reversion. (See Fig. 2-1) These expected cash benefits are then discounted at the appropriate rate to obtain the market value estimate.

Although these calculations are uncomplicated, they depend on the appraiser's assumptions or estimates. When using direct capitalization, the property's stabilized NOI must be estimated. This estimate is developed from market data (rental rates, vacancy and collection loss rates and operating expense data) for comparable properties in the market area; it represents the appraiser's estimation of how the property should perform.

Because the appraiser's opinion is based on observed market data, it's difficult to object to the NOI estimate. When a market is "normal," the idea of stabilized NOI is particularly useful.

However, there are two areas of concern. One: what if the property has significant vacancy at the time of the appraisal? Clearly, no one develops a property with the expectation of significant, permanent vacancy. So, the appraiser may use a market vacancy rate (or the anticipated vacancy rate when the property is fully leased) rather than the property's actual vacancy rate. Two: if the property's future NOI is expected to increase because of greater demand for space that leads to higher rental rates, direct capitalization of a single year's NOI may understate the property's value.

Because DCF analysis permits annual adjustments in rental rates, vacancy and collection loss rates and operating expenses, DCF analysis can be used to reflect a buyer's expectations of increasing NOI over time. When a property is expected to become fully leased during the next three to five years, for example, the current vacancy rate can be reduced until the desired occupancy is reached.
Accurate forecasting of expected changes results in realistic NOI estimates throughout the holding period—a result far superior to capitalizing a single year's NOI.

DCF analysis is ideally suited for situations such as this. However, DCF analysis does not add much useful information when the subject property is fully leased and no changes in occupancy or the external environment are anticipated that will affect NOI. As shown in Figure 2-1, the present value of a series of equal annual cash flows is the same as the capitalized value.

It is not an error to use a DCF analysis to value a property when no significant change in NOI is expected. However, it should be understood that this technique's results are no better than those produced by a correct application of direct capitalization. Of course, if both methods, properly used, result in similar values, the estimated value has greater credibility.

Figure 2-1 Comparison – Direct Cap and DCF

Figure 2-1. Estimating Value with Direct Capitalization and DCF Analysis

Direct Cap:

$$\text{Value} = \frac{\text{NOI}}{\text{Capitalization rate}} = \frac{\$10{,}000}{.06} = \$166{,}667$$

DCF (Yield Cap)

Year	Income	Reversion	Discount Rate	Present Value
1.	$10,000		0.943396	$9,434
2.	$10,000		0.889996	$8,900
3.	$10,000		0.839619	$8,396
4.	$10,000		0.792094	$7,921
5.	$10,000	$166,667	0.747258	$132,016
Present value of Cash Flows at 6%				$166,667

MORE ON YIELD CAPITALIZATION

Overall Yield Rate (Y_o): The rate of return on total capital invested, including both equity and debt. The Y_o takes into consideration changes in net income over the investment period and net reversion at the end of the holding period; it is applied to cash flow before debt service. It is also called the Property Yield Rate.

1. The income over the holding period is a series of incomes.
2. Reversion provides for recapture when using discounting but not in direct capitalization.

Equity Yield Rate (Y_E): A rate of return on equity capital as distinguished from the rate of return on debt capital; the equity is the investor's internal rate of return. The equity yield rate considers the effect of debt financing on the cash flow to the equity investor.

1. A true annualized rate of return on equity capital including (in addition to the average annual dividend) the full effect of any gain or loss from resale at the termination of the investment; a single discount rate that will make the present worth of all the future equity benefits equal the equity investment.

Equity dividend rate — Uses before-tax cash flow and is often referred to as cash-on-cash return.

Calculated by (net operating income - debt service) / equity investment.
Equity investment is the amount of money that the investor must contribute to purchase a property.

The equity dividend rate is also called the equity capitalization rate. Internal Rate of Return (IRR) - The annualized yield rate on capital that is (or could be) generated by an asset or within a group of capital assets over a period of ownership.

Internal Rate of Return is the rate that discounts all future cash flows to a net present worth equal to the original investment.

DISCOUNTED CASH FLOW (DCF)

1. A yield capitalization method used to calculate the present value of anticipated future cash flows.
2. Isolates differences in the timing of cash flows. Net cash flows from all time intervals, in the analysis are discounted to present value by an appropriate discount rate.
3. Using, data similar in investment holding period, net income estimates and financial terms, DCF should produce estimates similar to direct capitalization.

Yield capitalization

1. Yield capitalization is used with DCF, another method for converting income into value.

2. Yield methods require certain assumptions that vary. The assumptions include estimating such factors as the required rate of return on investment, the remaining economic life of the property, an investment holding period, and the income path. Anticipated depreciation or appreciation are reflected in the reversionary value.

3. It is a year-by-year breakdown of future benefits (net operating income) an investor expects to receive, discounting each year of the future benefit to a present value as of the appraisal date.

4. The net operating income that will be used for the annual discounting is normally after the deduction of property taxes. However, as the ad valorem process involves estimating value for tax purposes, this would almost appear to be accepting that the taxes were correct and thus the previous value must also be correct. DCF is looking at value through an investor's viewpoint and property taxes are an investment consideration. A method to handle taxes will be shown later.

5. One time that real estate taxes may be shown as an expense would be when the taxes are an expense stop for the owner of the property. At that point, the taxes are a fixed expense. This can occur normally in a lease but is sometimes seen in new leases just prior to or immediately after a reappraisal where property values have not been updated in a long period of time.

6. The process also converts the reversionary value into a present value. Reversion is the right of possession commencing on the termination of a particular estate. A simpler definition is when you sell the property at the end of the holding period and your estate (ownership) is no longer the property, but you have reverted, or converted, the asset back to cash.

REVIEW QUESTIONS

1. You are offered an annuity that will pay $17,000 per year for 7 years. If you feel that an appropriate discount rate is 10%, what is the annuity worth to you today? _____

2. The time value of money principle holds that, a _____ received _____ is worth _____ than a _____ to be _____ in the _____.

3. Yield capitalization is a more accurate appraisal methodology than direct capitalization. True / False

4. Frank will retire in 20 years. This year he wants to fund an amount of $150,000 to become available in 20 years. How much does he have to deposit into a pension plan earning 7% annually? _____

5. If the discount (or interest) rate is positive, the future value of an expected series of payments will always exceed the present value. True / False

6. The method of converting future net benefits into present value where each future net benefit is discounted at a proper yield rate is known as _____ .

7. The monthly payment on a mortgage of $250,000 at 8% interest over a 20-year term is _____.

8. Based on the information in the previous question, the annual debt service is _____ .

9. An investor expects to receive the following annual net incomes:

Year 1	$45,000
Year 2	$45,000
Year 3	$45,000
Year 4	$45,000
Year 5	$45,000

At the end of year five the property's reversion is expected to be $430,000. What is the present value of this five-year income stream, assuming a yield rate of 8%? _____

10. For amortization to occur, the annual mortgage constant must be _____ _____ than the interest rate.

11. If the investment in the property is not totally recaptured between the income stream and the reversion, then the yield rate is actually _____.

12. _____ capitalization is a method of converting an estimate of a _____ _____ income into value in one direct step.

13. What is the present value of the following income streams?

Year 1	$40,000
Year 2	$51,000
Year 3	$63,000
Year 4	$49,500

Assume an 8% discount rate is appropriate. _____

14. Given a rate of return of 10%, how much would I have to invest annually to accumulate $75,000 in 15 years? _____

15. The future worth of $1 is the _____of the present worth of $1.

Section 3
Additional Thoughts

Additional Thoughts

More on Investments

Net present value is the present value of the cash flows at the required rate of return of your project compared to your initial investment. In practical terms, it's a method of calculating your return on investment, or ROI, for a project or expenditure. By looking at all of the money you expect to make from the investment and translating those returns into today's dollars, you can decide whether the project is worthwhile.

Care must be used when developing the NPV, it is a calculation that is based on several assumptions and estimates, which means there's lots of room for error.

A net present value analysis is part of a process called Capital Budgeting. Capital budgeting is the process an investor or business undertakes to evaluate potential major projects or investments to determine which ones are worthwhile.

Example 3-1

You are looking at a new project and have estimated the following cash flows:

- Year 0: CF = -165,000
- Year 1: CF = 63,120
- Year 2: CF = 70,800
- Year 3: CF = 91,080

Your required return for assets of this risk is 12%.

You apply the 12% discount rate and determine the following:

Year 0:	(165,000)	1.000000	(165,000)
Year 1:	63,120	0.892857	56,357
Year 2:	70,800	0.797194	56,441
Year 3:	91,080	0.711780	64,829
		Net Present Value	12,627

The resulting positive net present value indicates this is a viable project.

Example 3-2

The project below is estimated to produce the following cash flows:

	Cash Flow
Year 0	(2,000,000)
Year 1	300,000
Year 2	350,000
Year 3	350,000
Year 4	350,000
Year 5	350,000
Year 6	350,000
Year 7	350,000
Year 8	375,000
Year 9	375,000
Year 10	375,000

The required return for assets of this risk is 12%.

Does the resulting net present value indicate this is not a viable project?

Year	Cash Flow	Discount Rate	Present Value
0	(2,000,000)	1.000000	(2,000,000)
1	300,000	0.892857	267,857
2	350,000	0.797194	279,018
3	350,000	0.711780	249,123
4	350,000	0.635518	222,431
5	350,000	0.567427	198,599
6	350,000	0.506631	177,321
7	350,000	0.452349	158,322
8	375,000	0.403883	151,456
9	375,000	0.360610	135,229
10	375,000	0.321973	120,740
		Net Present Value	(39,903)

Internal Rate of Return (IRR)

Example 3-3

Suppose the following information relates to a property you are valuing:

Initial Investment	$90,000
Required Rate of Return	10%
Reversion	$100,000

Using the cash flows below, what would be the internal rate of return (IRR)?

Year		Cash Flow
0		(90,000)
1		5,000
2		25,000
3		15,000
4		3,000
5		10,000
5		100,000

A financial calculator and excel will be used to demonstrate the example above.

Investment analysis

DCF can also be described as a valuation method used to estimate the attractiveness of an investment.

- This is a tool and not a guarantee. Small changes in inputs such as rent, expenses, capitalization rate and reversion can result in large value changes.
- Meaningful valuations depend on the users' ability to make solid cash flow projections. A single, unexpected event can immediately make a DCF model obsolete.

- There are properties that are truly investment type properties in which yield capitalization and the application of DCF should be considered.
- Often we hear comments about investment grade properties. These are generally good quality structures that are rented on short-term leases such as 5-10 years, have renewal options and are newer improvements. Examples of these would be:

    a. Office buildings
    b. Strip centers
    c. Shopping malls
    d. Apartment buildings could also be valued by DCF, although the leases are commonly month-to-month or on an annual basis.
    e. Office buildings leased to a government agency.

Other potential applications of DCF may be property that is:

a. Currently in a rent up period, such as a new property.
b. Property that is suffering from high vacancy with an expected increase in demand for the space.
c. An older property suffering from high vacancy under-going remodeling and updating to meet an anticipated need.
d. Contaminated properties with multiple-year clean-up costs.

DCF also lends itself to application on some unique properties such as mineral quarries, mines, landfills, and land development.

Older properties that are being leased on a long term could also be candidates for the application of DCF. However, buildings that are nearing the end of their economic life would not be a candidate for using DCF. The longer the DCF projection, the less reliable the answer generally will be.

A typical investor is going to analyze the past, current and future performance of a property prior to investing. In order to measure the response of the investor, DCF should be considered.

Investors analyzing the performance are similar to stocks or mutual funds. Analyses give you the last quarter, year-to-date, one- year, 5-

year and life of the fund performance to help you determine if it is a good investment.

DCF permits annual adjustments to rent, vacancy and expenses to reflect a buyer‘s expectations of increasing or decreasing net operating income over time. When a property is going to reduce the vacancy over a few years, the current vacancy rate can be reduced until the desired occupancy can be reached. ln direct capitalization, we use one year based upon historical data. But the income approach definition talks about anticipated future benefits.

A weakness of DCF is the reliance upon forecasting what is going to happen to the income stream, expenses and what the value of the property will be at the end of the holding period. Some of the considerations in making the forecasts are:

1. Current rent versus market rent and rent adjustments within the contract. This is much easier if we have the subject rent to compare to others in the determination if the rent is at market level.
2. Renewal options with any escalation provisions.
3. Anticipated tenant turnover. There is bound to be some turnover unless the contract coincides with the holding period. You may need to anticipate some vacancy after the conclusion of a contract to allow for tenant improvements prior to a new tenant.
4. Operating expenses that are the responsibility of the owner and consideration of any pass-through expenses to the tenant.
5. Changes in the Consumer Price Index that may affect the rent or expenses.
6. Residual value of the property at the end of the holding period and any associated selling or transaction costs.

Application of DCF can be summarized as follows:

1. Select a holding period for the investment. Normally the holding period will be for 5, 10 or 15 years with 10 being the most typically used.
2. Forecast all future gross and net cash flows.
3. Select an appropriate yield rate.

4. Convert the future benefits of each year's net cash flow and the reversion to a present value.

Quantity, variability, timing and duration of the cash flows

1. The quantity of the rent is simply how much is to be received.
2. Variability is how much change or dispersion there is in the rent from year to year.
    a. Rent may be received as an annuity.
    b. Annuity is the right to receive money or its equivalent in (usually) fixed equal amounts and at regular intervals for a definite or indefinite term.
    c. An annuity can be level, increasing or decreasing and does not have to be annual, so long as the amounts are scheduled or can be forecast.
    d. If an annuity in not level, it is referred to as a variable annuity.
    e. If the annuity (rent) is decreasing, the property may be nearing the end of its economic life. Once a property reaches the end of its economic life, it no longer contributes value and generally is not a candidate for DCF.
    f. If the net operating income is the same each year then this is referred to as a constant or level terminal annuity. lt is a terminal annuity because the income stream is going to stop at the end of the holding period.
    g. Real estate rent or income would be considered an annuity.

Timing of the income stream is when the rent will be received, whether it is all at the beginning of the year, monthly or at year end.

Income received at the end of each period is referred to as an ordinary annuity.

Typically, the rent is received monthly and the income is recognized within DCF as occurring at year end.

A lease that calls for the payments to be made at the beginning of each month creates an annuity in advance.

Duration of the income stream is how long it will last. That is until the end of the holding period.

Estimation of PGI Potential Gross Income (PGI) must be calculated for the subject property. As DCF uses several years, typically 5-10 as the basis for determining value, income must be recognized for each year.

Historical data for a property can also help to develop new rents by recognizing rent increases.

Normally a lease will set a rent for the base year and either state the rent amount for each year of the lease or it will state something to the effect of annual rent increases by 3% compounded annually.

In order to develop a DCF for a property, a percentage of change should be recognized within the calculation that is shown in the lease or is reasonable in relationship to similar properties.

In mass appraisal, leases are not often provided or accessible. The building of an in- house income data bank may help in determining typical rent increases in the marketplace plus typical escalation periods.

Reversionary rate

1. It is generally recognized that the rate to calculate the reversion may be different (higher) than the one used for the income stream.
2. The reversion occurs at the end of the holding period, this requires a longer projection period than most of the income stream, a higher degree of risk and a reduced remaining economic life of the improvement.
3. The income stream is often easier to project, especially with a long-term contract, than what the selling price of the subject property would be say 10 years from the date of the appraisal.
4. The rate used to calculate the reversionary value is sometimes referred to as the terminal rate (R_n).

Reversion

The proceeds of resale (the reversion) are accounted for in the final year of the analysis. The estimated reversion is often calculated by capitalizing the net

income estimated from the year immediately following the projection period. For example, in a 10-year analysis, the eleventh-year net income is projected and capitalized. An estimate of sale costs is deducted from the forecasted sale price to arrive at the net proceeds of resale. The capitalization rate used in this part of the analysis has many labels: residual cap rate, going-out cap rate, terminal cap rate or reversionary cap rate. The symbol is R_N. It is often greater than the going-in cap rate to account for the reduction in remaining economic life and added risk associated with the forecasted resale proceeds.

Each annual cash flow, including the reversion, is discounted to a present worth by way of a factor predicated upon a yield rate applicable to the problem. If the purpose is to estimate market value, the discount, or yield rate is market derived. Finally, each discounted cash flow is summed to an indication of the property's present value.

The reversion represents the return of the capital investment that was made in the property. The recapture of the investment may be received in the income stream and the reversion, or it may totally be received in the reversion.
If the capitalization rate on a purchased property is low, the investor may be stating they are looking to cover cost and return on the appreciation of the property at reversion. If the investment is not totally recaptured between the income stream and the reversion, then the yield rate is actually negative.
Typically, there would be some expenses associated with the reversion if the investor is going to sell at the end of the investment period.

- These expenses may be items such as brokerage fees, commissions, and legal fees.
- These expenses should be subtracted from the reversion prior to discounting to present value.

Appendix

Glossary

Glossary and Rates and Relationships

IRV

$I = R \times V$
$R = I \div V$
$V = I \div R$

VIF

$V = I \times F$
$I = V \div F$
$F = V \div I$

EAT

$E = A \times T$
$A = E \div T$
$T = E \div A$

$R_O = Y_O - CR$
$Y_O = R_O + CR$
$CR = Y_O - R_O$

$DCR = NOI \div I_M$

$R_O = DCR \times M \times R_M$

$R_O = NIR \div EGIM$

$R_O = M \times R_M + (1 - M) \times R_E$

$R_L = I_L \div V_L$

$R_B = I_B \div V_B$

$R_E = I_E \div V_E$

$R_M = I_M \div V_M$

$NIR = 1 - OER$

$OER = 1 - NIR$

$GIM = \text{Property Value} \div \text{Gross Income}$

$EGIM = \text{Property Value} \div \text{Effective Gross Income}$

Description of Various Rates

Overall Capitalization Rate (R_O) – The R_O reflects the relationship between a single years's net operating income and the total property value. It is typically derived from market comparisons with highly comparable sales, band of investment calculations, the DCR technique, NIR ÷ GIM, yield change techniques. It also reflects the weighted average of the R_L and R_B weighted by the respective contributions to value of the land and building. R_O can be either greater than, equal to, or less than Y_O depending on whether the property value is expected to decrease, remain stable, or increase respectively.

Overall Yield Rate (Y_O) – Sometimes referred to as the property discount rate or "r" or the "return on" rate, internal rate of return, or property interest rate. The overall yield rate is expressed as a compound annual percentage rate. It considers all expected benefits of property ownership, including the proceeds from the sale at the termination of the investment. It can be obtained by extraction from sales of similar properties, comparison with alternative investments of comparable risk, surveys of property investors, or the yield change formula – $Y_O = R_O + CR$. [CAUTION: Using the band of investment technique to blend Y_M and Y_E is mathematically incorrect unless the loan is "interest only" and there is no change in income or value over the holding period. Conceptually, the Y_O is a weighted average of the equity yield rate and the mortgage interest rate. However, in practice the Y_O is not usually calculated by the BOI technique because the ratio of debt and equity changes each year as the loan is amortized and the property value changes.]

Land Capitalization Rate (R_L) – The R_L reflects the relationship between a single year's net income attributable to the land and the value of the land. It is usually extracted from sales using the relationship $R_L = I_L \div V_L$. R_L can be either greater than, equal to, or less than Y_L depending on whether the land value is expected to decrease, remain stable, or increase respectively. [Note: Within the context of classic straight line capitalization where land income and value are not expected to change over the holding period, the land capitalization rate (including a property tax component) is the sum of the Y_O and the effective tax rate (because in that situation $R_L = Y_L$).]

Land Yield Rate (Y_L) – The Y_L reflects the rate of return on capital invested in the land portion of real property. It is expressed as a compound annual rate and considers all expected benefits of land ownership, including the proceeds from the sale of the land at the termination of the investment. Since $Y_L = Y_O = Y_B$ is virtually always true (because investors rarely assign a different risk rate to the land and building), the same methods used to derive Y_O are valid for deriving Y_L.

Building (Improvement) Capitalization Rate (R_B) – The R_B reflects the relationship between a single year's net income attributable to the building and the value of the building. It is usually extracted from sales using the relationship $R_B = I_B \div V_B$. R_B can be either greater than, equal to, or less than Y_B depending on whether the building value is expected to decrease, remain stable, or increase respectively. [Note: Within the context of classic straight line capitalization where building "return on" income is earned only on the undepreciated balance of the building investment during the holding period, the building capitalization rate (including a property tax component) is the sum of the Y_O, the recapture rate, and the effective tax rate (because in that situation, $R_B = Y_B$).]

Building (Improvement) Yield Rate (Y_B) – The Y_B reflects the rate of return on capital invested in the building portion of real property. It is expressed as a compound annual rate and considers all expected benefits of building ownership, including the proceeds from the sale of the building at the termination of the investment. Since $Y_B = Y_O = Y_L$ is virtually always true (because investors rarely assign a different risk rate to the land and building), the same methods used to derive Y_O are valid for deriving Y_B.

Mortgage Capitalization Rate (R_M) – Also known as the mortgage annual constant, the R_M is the ratio of the annual debt service (principal & interest). It reflects the relationship between mortgage income (I_M) and mortgage principal. It is one of the components used in the band of investment technique to get an R_O. It can be obtained from the compound interest tables by multiplying the partial payment factor (Column #6) by the number of conversion periods in one year.

Mortgage Yield Rate (Y_M) – The Y_M is also known as the mortgage interest rate. It is the lender's rate of return on a loan (if no points or other charges increase the lender's yield). It is usually obtained by interviewing lenders or can be extracted from the terms of the mortgage.

Equity Capitalization Rate (R_E) – The R_E is also known as the equity dividend rate or the cash on cash rate. It reflects the relationship between a single year's equity income and the value of the equity. It is typically extracted from sales using the relationship $R_E = I_E \div V_E$. It is extremely sensitive to loan-to-value ratios. [CAUTION: Unlike the Y_E, the R_E should not be derived from sales of stocks or bonds because of dissimilarity of investments, i.e. liquidity, holding term, etc.]

Equity Yield Rate (Y_E) – The Y_E is the "return on" rate for equity and is sometimes referred to as the equity discount rate or the equity interest rate. It considers all the expected cash flows attributable to the equity investment, including proceeds from sale at the termination of the investment. The Y_E can be extracted from sales of similar properties, from alternative investments of comparable risk such as stocks and bonds, or from surveys of market participants.

Loan to Value Ratio (M) – This is the ratio between a mortgage loan and the value of the property, usually expressed as a percentage. It is usually obtained from interviews with active lenders. It can be extracted from sales by dividing the loan amount by the selling price.

Debt Coverage Ratio (DCR) – This is the ratio of net operating income to annual debt service (principal & interest). It measures the ability of a property to meet its annual debt requirements out of net operating income. The DCR can be extracted from sales by dividing the net operating income by the annual debt service. The DCR is an ideal test of reasonableness and can be multiplied by the R_M and the M to obtain R_O. ($R_O = DCR \times R_M \times M$)

Effective Tax Rate (ETR) – This is the ratio of property taxes to total property value. It can be derived by the EAT formula by multiplying the assessment level times the tax rate as long as all components are expressed as proper decimals. ($ETR = A \times T$) It can also be extracted from sales by dividing the income to property taxes by total property value. It can be directly added to or subtracted from the R_O to get an R_O with or without a property tax component. Conceptually, the ETR cannot be directly added to or subtracted from the Y_O to get a Y_O with or without a property tax component because Y_O is a yield rate used in discounting the cash flows in yield capitalization and the ETR is a capitalization rate. An exception to this is when using classic straight line capitalization where the $Y_O = R_L$.

Operating Expense Ratio (OER) – The OER reflects the relationship between operating expenses and the Effective Gross Income (EGI) or between the relationship between operating expenses and Potential Gross Income. Consistency is paramount to using this ratio properly. If the OER is represented to be a percentage of EGI it must be applied consistently with the manner in which it was derived. Likewise, if the OER is represented to be a percentage of Potential Gross Income (PGI) it must be applied consistently with the manner in which it was derived.

Net Income Ratio (NIR) – The NIR reflects the relationship between net operating income and the Effective Gross Income (EGI) or between the relationship between net operating income and Potential Gross Income (PGI). Consistency is paramount to using this ratio properly. If the NIR is represented to be a percentage of EGI it must be applied consistently with the manner in which it was derived. Likewise, if the NIR is represented to be a percentage of (PGI) it must be applied consistently with the manner in which it was derived.

Abbreviations Used in Income Approach to Value

BOI	Band of Investment
CR	Constant Rate of Change
DCR	Debt Coverage Ratio
EAT	Effective Tax Rate, Assessment Level, & Tax Rate formula
EGI	Effective Gross Income
EGIM	Effective Gross Income Multiplier
ETR	Effective Tax Rate
GIM	Gross Income Multiplier (sometimes known as Gross Rent Multiplier)
GLA	Gross Leasable Area
GRM	Gross Rent Multiplier (sometimes known as Gross Income Multiplier)
I_B	Income to the Building (Improvement)
I_E	Income to the Equity
I_L	Income to the Land
I_M	Income to the Mortgage (income necessary to pay principal & interest)
IRV	Income Rate & Value formula
I_V	Income to total property value (Net Operating Income)
NIR	Net Income Ratio (also known as Net Operating Income Ratio or 1 - OER)
NLA	Net Leasable Area
NOI	Net Operating Income
OER	Operating Expense Ratio (or 1 - NIR)
PAV	Property Assessment Valuation (published 1996)
PGI	Potential Gross Income
PW	Present Worth
R_B	Building Capitalization Rate (also known as the Improvement Capitalization Rate)
R_E	Equity Capitalization Rate (also known as the Equity Dividend Rate or Cash on Cash Rate)
R_L	Land Capitalization Rate
R_M	Mortgage Capitalization Rate
R_O	Overall Capitalization Rate (also known as OAR)
V_B	Value of the Building (Improvement)
V_E	Value of Equity
VIF	Value Income & Factor formula
V_L	Value of the Land
V_M	Value of the Mortgage (Mortgage Principal)
V&C	Vacancy & Collection Loss
Y_B	Building Yield Rate
Y_E	Equity Yield Rate
Y_L	Land Yield Rate
Y_M	Mortgage Yield Rate (Mortgage Interest Rate or lender's yield on the loan)
Y_O	Overall Yield Rate (also known as the Discount Rate or "r")

Compound Interest Tables

Compound Interest Tables

Annual Compound Interest tables follow:

6% Annual Compound Interest Table

Period	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.060000	1.000000	1.000000	0.943396	0.943396	1.06000000
2	1.123600	2.060000	0.485437	0.889996	1.833393	0.54543689
3	1.191016	3.183600	0.314110	0.839619	2.673012	0.37410981
4	1.262477	4.374616	0.228591	0.792094	3.465106	0.28859149
5	1.338226	5.637093	0.177396	0.747258	4.212364	0.23739640
6	1.418519	6.975319	0.143363	0.704961	4.917324	0.20336263
7	1.503630	8.393838	0.119135	0.665057	5.582381	0.17913502
8	1.593848	9.897468	0.101036	0.627412	6.209794	0.16103594
9	1.689479	11.491316	0.087022	0.591898	6.801692	0.14702224
10	1.790848	13.180795	0.075868	0.558395	7.360087	0.13586796
11	1.898299	14.971643	0.066793	0.526788	7.886875	0.12679294
12	2.012196	16.869941	0.059277	0.496969	8.383844	0.11927703
13	2.132928	18.882138	0.052960	0.468839	8.852683	0.11296011
14	2.260904	21.015066	0.047585	0.442301	9.294984	0.10758491
15	2.396558	23.275970	0.042963	0.417265	9.712249	0.10296276
16	2.540352	25.672528	0.038952	0.393646	10.105895	0.09895214
17	2.692773	28.212880	0.035445	0.371364	10.477260	0.09544480
18	2.854339	30.905653	0.032357	0.350344	10.827603	0.09235654
19	3.025600	33.759992	0.029621	0.330513	11.158116	0.08962086
20	3.207135	36.785591	0.027185	0.311805	11.469921	0.08718456
21	3.399564	39.992727	0.025005	0.294155	11.764077	0.08500455
22	3.603537	43.392290	0.023046	0.277505	12.041582	0.08304557
23	3.819750	46.995828	0.021278	0.261797	12.303379	0.08127848
24	4.048935	50.815577	0.019679	0.246979	12.550358	0.07967900
25	4.291871	54.864512	0.018227	0.232999	12.783356	0.07822672
26	4.549383	59.156383	0.016904	0.219810	13.003166	0.07690435
27	4.822346	63.705766	0.015697	0.207368	13.210534	0.07569717
28	5.111687	68.528112	0.014593	0.195630	13.406164	0.07459255
29	5.418388	73.639798	0.013580	0.184557	13.590721	0.07357961
30	5.743491	79.058186	0.012649	0.174110	13.764831	0.07264891
31	6.088101	84.801677	0.011792	0.164255	13.929086	0.07179222
32	6.453387	90.889778	0.011002	0.154957	14.084043	0.07100234
33	6.840590	97.343165	0.010273	0.146186	14.230230	0.07027293
34	7.251025	104.183755	0.009598	0.137912	14.368141	0.06959843
35	7.686087	111.434780	0.008974	0.130105	14.498246	0.06897386
36	8.147252	119.120867	0.008395	0.122741	14.620987	0.06839483
37	8.636087	127.268119	0.007857	0.115793	14.736780	0.06785743
38	9.154252	135.904206	0.007358	0.109239	14.846019	0.06735812
39	9.703507	145.058458	0.006894	0.103056	14.949075	0.06689377
40	10.285718	154.761966	0.006462	0.097222	15.046297	0.06646154

6% Monthly Compound Interest Table

Month	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.005000	1.000000	1.000000	0.995025	0.995025	1.00500000
2	1.010025	2.005000	0.498753	0.990075	1.985099	0.50375312
3	1.015075	3.015025	0.331672	0.985149	2.970248	0.33667221
4	1.020151	4.030100	0.248133	0.980248	3.950496	0.25313279
5	1.025251	5.050251	0.198010	0.975371	4.925866	0.20300997
6	1.030378	6.075502	0.164595	0.970518	5.896384	0.16959546
7	1.035529	7.105879	0.140729	0.965690	6.862074	0.14572854
8	1.040707	8.141409	0.122829	0.960885	7.822959	0.12782886
9	1.045911	9.182116	0.108907	0.956105	8.779064	0.11390736
10	1.051140	10.228026	0.097771	0.951348	9.730412	0.10277057
11	1.056396	11.279167	0.088659	0.946615	10.677027	0.09365903
12	1.061678	12.335562	0.081066	0.941905	11.618932	0.08606643
Year						
1	1.061678	12.335562	0.081066	0.941905	11.618932	0.08606643
2	1.127160	25.431955	0.039321	0.887186	22.562866	0.04432061
3	1.196681	39.336105	0.025422	0.835645	32.871016	0.03042194
4	1.270489	54.097832	0.018485	0.787098	42.580318	0.02348503
5	1.348850	69.770031	0.014333	0.741372	51.725561	0.01933280
6	1.432044	86.408856	0.011573	0.698302	60.339514	0.01657289
7	1.520370	104.073927	0.009609	0.657735	68.453042	0.01460855
8	1.614143	122.828542	0.008141	0.619524	76.095218	0.01314143
9	1.713699	142.739900	0.007006	0.583533	83.293424	0.01200575
10	1.819397	163.879347	0.006102	0.549633	90.073453	0.01110205
11	1.931613	186.322629	0.005367	0.517702	96.459599	0.01036703
12	2.050751	210.150163	0.004759	0.487626	102.474743	0.00975850
13	2.177237	235.447328	0.004247	0.459298	108.140440	0.00924723
14	2.311524	262.304766	0.003812	0.432615	113.476990	0.00881236
15	2.454094	290.818712	0.003439	0.407482	118.503515	0.00843857
16	2.605457	321.091337	0.003114	0.383810	123.238025	0.00811438
17	2.766156	353.231110	0.002831	0.361513	127.697486	0.00783101
18	2.936766	387.353194	0.002582	0.340511	131.897876	0.00758162
19	3.117899	423.579854	0.002361	0.320729	135.854246	0.00736083
20	3.310204	462.040895	0.002164	0.302096	139.580772	0.00716431
21	3.514371	502.874129	0.001989	0.284546	143.090806	0.00698857
22	3.731129	546.225867	0.001831	0.268015	146.396927	0.00683074
23	3.961257	592.251446	0.001688	0.252445	149.510979	0.00668847
24	4.205579	641.115782	0.001560	0.237779	152.444121	0.00655978
25	4.464970	692.993962	0.001443	0.223966	155.206864	0.00644301
30	6.022575	1004.515042	0.000996	0.166042	166.791614	0.00599551
35	8.123551	1424.710299	0.000702	0.123099	175.380226	0.00570190
40	10.957454	1991.490734	0.000502	0.091262	181.747584	0.00550214

7% Annual Compound Interest Table

Period	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.070000	1.000000	1.000000	0.934579	0.934579	1.07000000
2	1.144900	2.070000	0.483092	0.873439	1.808018	0.55309179
3	1.225043	3.214900	0.311052	0.816298	2.624316	0.38105167
4	1.310796	4.439943	0.225228	0.762895	3.387211	0.29522812
5	1.402552	5.750739	0.173891	0.712986	4.100197	0.24389069
6	1.500730	7.153291	0.139796	0.666342	4.766540	0.20979580
7	1.605781	8.654021	0.115553	0.622750	5.389289	0.18555322
8	1.718186	10.259803	0.097468	0.582009	5.971299	0.16746776
9	1.838459	11.977989	0.083486	0.543934	6.515232	0.15348647
10	1.967151	13.816448	0.072378	0.508349	7.023582	0.14237750
11	2.104852	15.783599	0.063357	0.475093	7.498674	0.13335690
12	2.252192	17.888451	0.055902	0.444012	7.942686	0.12590199
13	2.409845	20.140643	0.049651	0.414964	8.357651	0.11965085
14	2.578534	22.550488	0.044345	0.387817	8.745468	0.11434494
15	2.759032	25.129022	0.039795	0.362446	9.107914	0.10979462
16	2.952164	27.888054	0.035858	0.338735	9.446649	0.10585765
17	3.158815	30.840217	0.032425	0.316574	9.763223	0.10242519
18	3.379932	33.999033	0.029413	0.295864	10.059087	0.09941260
19	3.616528	37.378965	0.026753	0.276508	10.335595	0.09675301
20	3.869684	40.995492	0.024393	0.258419	10.594014	0.09439293
21	4.140562	44.865177	0.022289	0.241513	10.835527	0.09228900
22	4.430402	49.005739	0.020406	0.225713	11.061240	0.09040577
23	4.740530	53.436141	0.018714	0.210947	11.272187	0.08871393
24	5.072367	58.176671	0.017189	0.197147	11.469334	0.08718902
25	5.427433	63.249038	0.015811	0.184249	11.653583	0.08581052
26	5.807353	68.676470	0.014561	0.172195	11.825779	0.08456103
27	6.213868	74.483823	0.013426	0.160930	11.986709	0.08342573
28	6.648838	80.697691	0.012392	0.150402	12.137111	0.08239193
29	7.114257	87.346529	0.011449	0.140563	12.277674	0.08144865
30	7.612255	94.460786	0.010586	0.131367	12.409041	0.08058640
31	8.145113	102.073041	0.009797	0.122773	12.531814	0.07979691
32	8.715271	110.218154	0.009073	0.114741	12.646555	0.07907292
33	9.325340	118.933425	0.008408	0.107235	12.753790	0.07840807
34	9.978114	128.258765	0.007797	0.100219	12.854009	0.07779674
35	10.676581	138.236878	0.007234	0.093663	12.947672	0.07723396
36	11.423942	148.913460	0.006715	0.087535	13.035208	0.07671531
37	12.223618	160.337402	0.006237	0.081809	13.117017	0.07623685
38	13.079271	172.561020	0.005795	0.076457	13.193473	0.07579505
39	13.994820	185.640292	0.005387	0.071455	13.264928	0.07538676
40	14.974458	199.635112	0.005009	0.066780	13.331709	0.07500914

7% Monthly Compound Interest Table

Month	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.005833	1.000000	1.000000	0.994200	0.994200	1.00583333
2	1.011701	2.005833	0.498546	0.988435	1.982635	0.50437924
3	1.017602	3.017534	0.331396	0.982702	2.965337	0.33722976
4	1.023538	4.035136	0.247823	0.977003	3.942340	0.25365644
5	1.029509	5.058675	0.197680	0.971337	4.913677	0.20351357
6	1.035514	6.088184	0.164253	0.965704	5.879381	0.17008594
7	1.041555	7.123698	0.140377	0.960103	6.839484	0.14620986
8	1.047631	8.165253	0.122470	0.954535	7.794019	0.12830352
9	1.053742	9.212883	0.108544	0.948999	8.743018	0.11437698
10	1.059889	10.266625	0.097403	0.943495	9.686513	0.10323632
11	1.066071	11.326514	0.088288	0.938024	10.624537	0.09412175
12	1.072290	12.392585	0.080693	0.932583	11.557120	0.08652675
Year						
1	1.072290	12.392585	0.080693	0.932583	11.557120	0.08652675
2	1.149806	25.681032	0.038939	0.869712	22.335099	0.04477258
3	1.232926	39.930101	0.025044	0.811079	32.386464	0.03087710
4	1.322054	55.209236	0.018113	0.756399	41.760201	0.02394624
5	1.417625	71.592902	0.013968	0.705405	50.501994	0.01980120
6	1.520106	89.160944	0.011216	0.657849	58.654444	0.01704901
7	1.629994	107.998981	0.009259	0.613499	66.257285	0.01509268
8	1.747826	128.198821	0.007800	0.572139	73.347569	0.01363372
9	1.874177	149.858909	0.006673	0.533568	79.959850	0.01250628
10	2.009661	173.084807	0.005778	0.497596	86.126354	0.01161085
11	2.154940	197.989707	0.005051	0.464050	91.877134	0.01088410
12	2.310721	224.694985	0.004450	0.432765	97.240216	0.01028381
13	2.477763	253.330789	0.003947	0.403590	102.241738	0.00978074
14	2.656881	284.036677	0.003521	0.376381	106.906074	0.00935401
15	2.848947	316.962297	0.003155	0.351007	111.255958	0.00898828
16	3.054897	352.268112	0.002839	0.327343	115.312587	0.00867208
17	3.275736	390.126188	0.002563	0.305275	119.095732	0.00839661
18	3.512539	430.721027	0.002322	0.284694	122.623831	0.00815502
19	3.766461	474.250470	0.002109	0.265501	125.914077	0.00794192
20	4.038739	520.926660	0.001920	0.247602	128.982506	0.00775299
21	4.330700	570.977075	0.001751	0.230910	131.844073	0.00758472
22	4.643766	624.645640	0.001601	0.215342	134.512723	0.00743424
23	4.979464	682.193909	0.001466	0.200825	137.001461	0.00729919
24	5.339430	743.902347	0.001344	0.187286	139.322418	0.00717760
25	5.725418	810.071693	0.001234	0.174660	141.486903	0.00706779
30	8.116497	1219.970996	0.000820	0.123206	150.307568	0.00665302
35	11.506152	1801.054601	0.000555	0.086910	156.529709	0.00638856
40	16.311411	2624.813398	0.000381	0.061307	160.918839	0.00621431

8% Annual Compound Interest Table

Period	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.080000	1.000000	1.000000	0.925926	0.925926	1.08000000
2	1.166400	2.080000	0.480769	0.857339	1.783265	0.56076923
3	1.259712	3.246400	0.308034	0.793832	2.577097	0.38803351
4	1.360489	4.506112	0.221921	0.735030	3.312127	0.30192080
5	1.469328	5.866601	0.170456	0.680583	3.992710	0.25045645
6	1.586874	7.335929	0.136315	0.630170	4.622880	0.21631539
7	1.713824	8.922803	0.112072	0.583490	5.206370	0.19207240
8	1.850930	10.636628	0.094015	0.540269	5.746639	0.17401476
9	1.999005	12.487558	0.080080	0.500249	6.246888	0.16007971
10	2.158925	14.486562	0.069029	0.463193	6.710081	0.14902949
11	2.331639	16.645487	0.060076	0.428883	7.138964	0.14007634
12	2.518170	18.977126	0.052695	0.397114	7.536078	0.13269502
13	2.719624	21.495297	0.046522	0.367698	7.903776	0.12652181
14	2.937194	24.214920	0.041297	0.340461	8.244237	0.12129685
15	3.172169	27.152114	0.036830	0.315242	8.559479	0.11682954
16	3.425943	30.324283	0.032977	0.291890	8.851369	0.11297687
17	3.700018	33.750226	0.029629	0.270269	9.121638	0.10962943
18	3.996019	37.450244	0.026702	0.250249	9.371887	0.10670210
19	4.315701	41.446263	0.024128	0.231712	9.603599	0.10412763
20	4.660957	45.761964	0.021852	0.214548	9.818147	0.10185221
21	5.033834	50.422921	0.019832	0.198656	10.016803	0.09983225
22	5.436540	55.456755	0.018032	0.183941	10.200744	0.09803207
23	5.871464	60.893296	0.016422	0.170315	10.371059	0.09642217
24	6.341181	66.764759	0.014978	0.157699	10.528758	0.09497796
25	6.848475	73.105940	0.013679	0.146018	10.674776	0.09367878
26	7.396353	79.954415	0.012507	0.135202	10.809978	0.09250713
27	7.988061	87.350768	0.011448	0.125187	10.935165	0.09144810
28	8.627106	95.338830	0.010489	0.115914	11.051078	0.09048891
29	9.317275	103.965936	0.009619	0.107328	11.158406	0.08961854
30	10.062657	113.283211	0.008827	0.099377	11.257783	0.08882743
31	10.867669	123.345868	0.008107	0.092016	11.349799	0.08810728
32	11.737083	134.213537	0.007451	0.085200	11.434999	0.08745081
33	12.676050	145.950620	0.006852	0.078889	11.513888	0.08685163
34	13.690134	158.626670	0.006304	0.073045	11.586934	0.08630411
35	14.785344	172.316804	0.005803	0.067635	11.654568	0.08580326
36	15.968172	187.102148	0.005345	0.062625	11.717193	0.08534467
37	17.245626	203.070320	0.004924	0.057986	11.775179	0.08492440
38	18.625276	220.315945	0.004539	0.053690	11.828869	0.08453894
39	20.115298	238.941221	0.004185	0.049713	11.878582	0.08418513
40	21.724521	259.056519	0.003860	0.046031	11.924613	0.08386016

8% Monthly Compound Interest Table

Month	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.006667	1.000000	1.000000	0.993377	0.993377	1.00666667
2	1.013378	2.006667	0.498339	0.986799	1.980176	0.50500554
3	1.020134	3.020044	0.331121	0.980264	2.960440	0.33778762
4	1.026935	4.040178	0.247514	0.973772	3.934212	0.25418051
5	1.033781	5.067113	0.197351	0.967323	4.901535	0.20401772
6	1.040673	6.100893	0.163910	0.960917	5.862452	0.17057709
7	1.047610	7.141566	0.140025	0.954553	6.817005	0.14669198
8	1.054595	8.189176	0.122112	0.948232	7.765237	0.12877907
9	1.061625	9.243771	0.108181	0.941952	8.707189	0.11484763
10	1.068703	10.305396	0.097037	0.935714	9.642903	0.10370321
11	1.075827	11.374099	0.087919	0.929517	10.572420	0.09458572
12	1.083000	12.449926	0.080322	0.923361	11.495782	0.08698843
Year						
1	1.083000	12.449926	0.080322	0.923361	11.495782	0.08698843
2	1.172888	25.933190	0.038561	0.852596	22.110544	0.04522729
3	1.270237	40.535558	0.024670	0.787255	31.911806	0.03133637
4	1.375666	56.349915	0.017746	0.726921	40.961913	0.02441292
5	1.489846	73.476856	0.013610	0.671210	49.318433	0.02027639
6	1.613502	92.025325	0.010867	0.619770	57.034522	0.01753324
7	1.747422	112.113308	0.008920	0.572272	64.159261	0.01558621
8	1.892457	133.868583	0.007470	0.528414	70.737970	0.01413668
9	2.049530	157.429535	0.006352	0.487917	76.812497	0.01301871
10	2.219640	182.946035	0.005466	0.450523	82.421481	0.01213276
11	2.403869	210.580392	0.004749	0.415996	87.600600	0.01141545
12	2.603389	240.508387	0.004158	0.384115	92.382800	0.01082453
13	2.819469	272.920390	0.003664	0.354677	96.798498	0.01033074
14	3.053484	308.022574	0.003247	0.327495	100.875784	0.00991318
15	3.306921	346.038222	0.002890	0.302396	104.640592	0.00955652
16	3.581394	387.209149	0.002583	0.279221	108.116871	0.00924925
17	3.878648	431.797244	0.002316	0.257822	111.326733	0.00898257
18	4.200574	480.086128	0.002083	0.238063	114.290596	0.00874963
19	4.549220	532.382966	0.001878	0.219818	117.027313	0.00854501
20	4.926803	589.020416	0.001698	0.202971	119.554292	0.00836440
21	5.335725	650.358746	0.001538	0.187416	121.887606	0.00820428
22	5.778588	716.788127	0.001395	0.173053	124.042099	0.00806178
23	6.258207	788.731114	0.001268	0.159790	126.031475	0.00793453
24	6.777636	866.645333	0.001154	0.147544	127.868388	0.00782054
25	7.340176	951.026395	0.001051	0.136237	129.564523	0.00771816
30	10.935730	1490.359449	0.000671	0.091443	136.283494	0.00733765
35	16.292550	2293.882485	0.000436	0.061378	140.793338	0.00710261
40	24.273386	3491.007831	0.000286	0.041197	143.820392	0.00695312

10% Annual Compound Interest Table

Period	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.100000	1.000000	1.000000	0.909091	0.909091	1.10000000
2	1.210000	2.100000	0.476190	0.826446	1.735537	0.57619048
3	1.331000	3.310000	0.302115	0.751315	2.486852	0.40211480
4	1.464100	4.641000	0.215471	0.683013	3.169865	0.31547080
5	1.610510	6.105100	0.163797	0.620921	3.790787	0.26379748
6	1.771561	7.715610	0.129607	0.564474	4.355261	0.22960738
7	1.948717	9.487171	0.105405	0.513158	4.868419	0.20540550
8	2.143589	11.435888	0.087444	0.466507	5.334926	0.18744402
9	2.357948	13.579477	0.073641	0.424098	5.759024	0.17364054
10	2.593742	15.937425	0.062745	0.385543	6.144567	0.16274539
11	2.853117	18.531167	0.053963	0.350494	6.495061	0.15396314
12	3.138428	21.384284	0.046763	0.318631	6.813692	0.14676332
13	3.452271	24.522712	0.040779	0.289664	7.103356	0.14077852
14	3.797498	27.974983	0.035746	0.263331	7.366687	0.13574622
15	4.177248	31.772482	0.031474	0.239392	7.606080	0.13147378
16	4.594973	35.949730	0.027817	0.217629	7.823709	0.12781662
17	5.054470	40.544703	0.024664	0.197845	8.021553	0.12466413
18	5.559917	45.599173	0.021930	0.179859	8.201412	0.12193022
19	6.115909	51.159090	0.019547	0.163508	8.364920	0.11954687
20	6.727500	57.274999	0.017460	0.148644	8.513564	0.11745962
21	7.400250	64.002499	0.015624	0.135131	8.648694	0.11562439
22	8.140275	71.402749	0.014005	0.122846	8.771540	0.11400506
23	8.954302	79.543024	0.012572	0.111678	8.883218	0.11257181
24	9.849733	88.497327	0.011300	0.101526	8.984744	0.11129978
25	10.834706	98.347059	0.010168	0.092296	9.077040	0.11016807
26	11.918177	109.181765	0.009159	0.083905	9.160945	0.10915904
27	13.109994	121.099942	0.008258	0.076278	9.237223	0.10825764
28	14.420994	134.209936	0.007451	0.069343	9.306567	0.10745101
29	15.863093	148.630930	0.006728	0.063039	9.369606	0.10672807
30	17.449402	164.494023	0.006079	0.057309	9.426914	0.10607925
31	19.194342	181.943425	0.005496	0.052099	9.479013	0.10549621
32	21.113777	201.137767	0.004972	0.047362	9.526376	0.10497172
33	23.225154	222.251544	0.004499	0.043057	9.569432	0.10449941
34	25.547670	245.476699	0.004074	0.039143	9.608575	0.10407371
35	28.102437	271.024368	0.003690	0.035584	9.644159	0.10368971
36	30.912681	299.126805	0.003343	0.032349	9.676508	0.10334306
37	34.003949	330.039486	0.003030	0.029408	9.705917	0.10302994
38	37.404343	364.043434	0.002747	0.026735	9.732651	0.10274692
39	41.144778	401.447778	0.002491	0.024304	9.756956	0.10249098
40	45.259256	442.592556	0.002259	0.022095	9.779051	0.10225941

10% Monthly Compound Interest Table

Month	FV of $1	FV of an Annuity of $1	Sinking Fund	PV of $1	PV of an Annuity of $1	Amount to Amortize $1
1	1.008333	1.000000	1.000000	0.991736	0.991736	1.00833333
2	1.016736	2.008333	0.497925	0.983539	1.975275	0.50625864
3	1.025209	3.025069	0.330571	0.975411	2.950686	0.33890426
4	1.033752	4.050278	0.246897	0.967350	3.918036	0.25522994
5	1.042367	5.084031	0.196694	0.959355	4.877391	0.20502766
6	1.051053	6.126398	0.163228	0.951427	5.828817	0.17156139
7	1.059812	7.177451	0.139325	0.943563	6.772381	0.14765856
8	1.068644	8.237263	0.121400	0.935765	7.708146	0.12973288
9	1.077549	9.305907	0.107459	0.928032	8.636178	0.11579196
10	1.086529	10.383456	0.096307	0.920362	9.556540	0.10464038
11	1.095583	11.469985	0.087184	0.912756	10.469296	0.09551741
12	1.104713	12.565568	0.079583	0.905212	11.374508	0.08791589
Year						
1	1.104713	12.565568	0.079583	0.905212	11.374508	0.08791589
2	1.220391	26.446915	0.037812	0.819410	21.670855	0.04614493
3	1.348182	41.781821	0.023934	0.741740	30.991236	0.03226719
4	1.489354	58.722492	0.017029	0.671432	39.428160	0.02536258
5	1.645309	77.437072	0.012914	0.607789	47.065369	0.02124704
6	1.817594	98.111314	0.010193	0.550178	53.978665	0.01852584
7	2.007920	120.950418	0.008268	0.498028	60.236667	0.01660118
8	2.218176	146.181076	0.006841	0.450821	65.901488	0.01517416
9	2.450448	174.053713	0.005745	0.408089	71.029355	0.01407869
10	2.707041	204.844979	0.004882	0.369407	75.671163	0.01321507
11	2.990504	238.860493	0.004187	0.334392	79.872986	0.01251988
12	3.303649	276.437876	0.003617	0.302696	83.676528	0.01195078
13	3.649584	317.950102	0.003145	0.274004	87.119542	0.01147848
14	4.031743	363.809201	0.002749	0.248032	90.236201	0.01108203
15	4.453920	414.470346	0.002413	0.224521	93.057439	0.01074605
16	4.920303	470.436376	0.002126	0.203240	95.611259	0.01045902
17	5.435523	532.262780	0.001879	0.183975	97.923008	0.01021210
18	6.004693	600.563216	0.001665	0.166536	100.015633	0.00999844
19	6.633463	676.015601	0.001479	0.150751	101.909902	0.00981259
20	7.328074	759.368836	0.001317	0.136462	103.624619	0.00965022
21	8.095419	851.450244	0.001174	0.123527	105.176801	0.00950780
22	8.943115	953.173779	0.001049	0.111818	106.581856	0.00938246
23	9.879576	1065.549097	0.000938	0.101219	107.853730	0.00927182
24	10.914097	1189.691580	0.000841	0.091625	109.005045	0.00917389
25	12.056945	1326.833403	0.000754	0.082940	110.047230	0.00908701
30	19.837399	2260.487925	0.000442	0.050410	113.950820	0.00877572
35	32.638650	3796.638052	0.000263	0.030639	116.323377	0.00859672
40	53.700663	6324.079581	0.000158	0.018622	117.765391	0.00849146